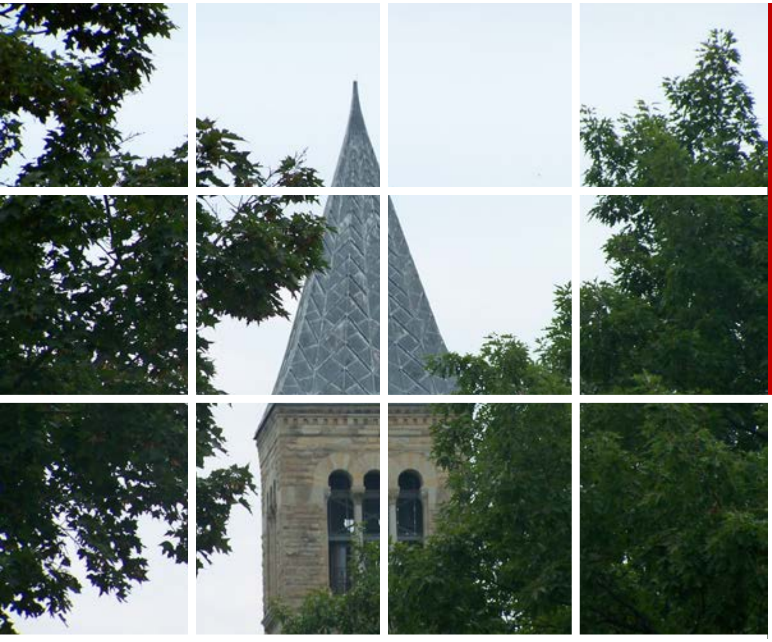


Vacuum Science and Technology for Accelerator Vacuum Systems

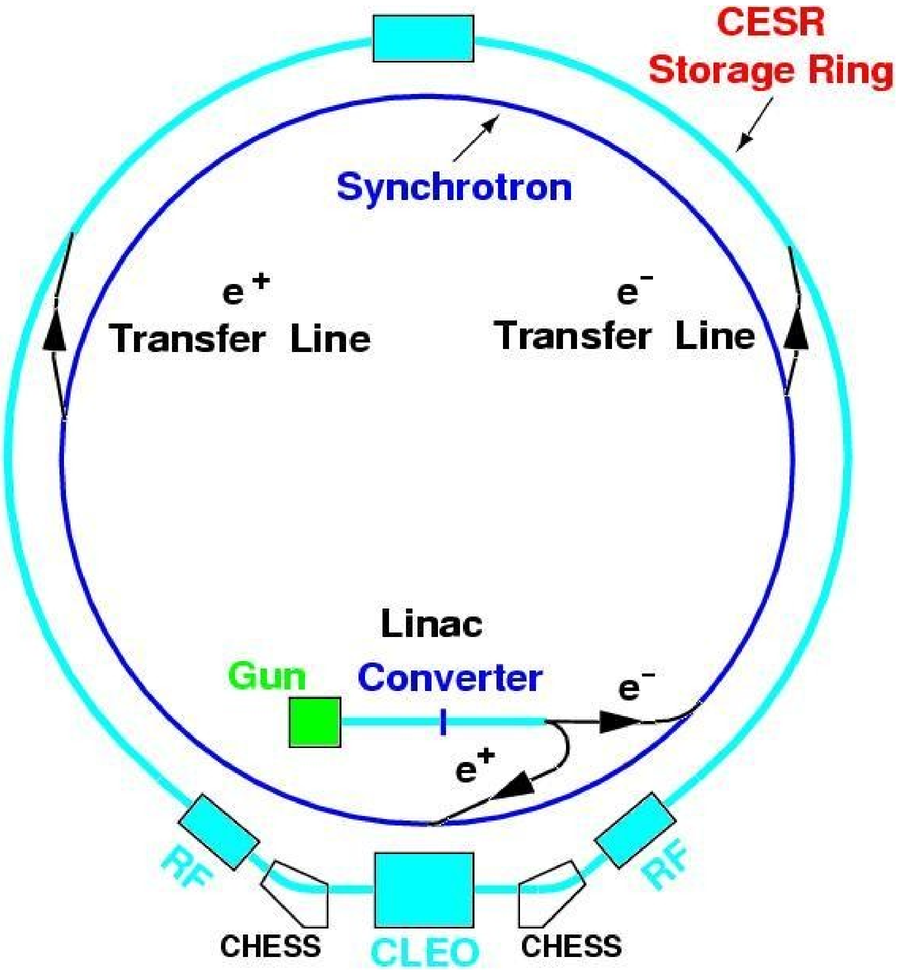
Yulin Li and Xianghong Liu
Cornell University, Ithaca, NY

UCDAVIS

Cornell Laboratory
for Accelerator-based Sciences
and Education (CLASSE)

About Your Instructors

- Yulin Li & Xianghong Liu
  - Vacuum Scientists (yulin.li@cornell.edu, XL66@cornell.edu)
  - Cornell Laboratory for Accelerator-based Sciences and Educations (CLASSE)
  - Self-taught vacuum practitioners, starting as surface scientists
  - As research staff members at CLASSE over 20+ & 10+ years overseeing all aspect of vacuum operations, designs and R&D
- CLASSE – A Research Center on Cornell's Ithaca Campus
  - Primary funding – NSF, with additional funding from DOE
  - Cornell Electron Storage Ring (CSER): 780-m, 500-mA, 5-GeV
  - Cornell High Energy Synchrotron Sources (CHESS) – A SR-user facility w/ 12 experimental stations
  - CesrTA – CESR as a Test Accelerator for International Linear Collider Damping Ring R&D, with focus on Electron Cloud studies
  - Cornell Energy Recover LINAC – DC Photo-cathode injector
  - Superconducting RF researches
  - High-energy physics – CLEO, LHC, etc.


CESR
Storage Ring
Synchrotron
e+
Transfer Line
e-
Transfer Line
Linac
Gun
Converter
e-
e+
RF
RF
CHESS
CLEO
CHESS

Acknowledgements

- *Special thanks go to Mr. Lou Bertolini of LLNL, who shared ~1000 slides with me from his previous USPAS Vacuum lectures.*

- *Thanks also go to many of my colleagues from other accelerator facilities, who provided valuable information.*

- *Many vacuum product companies graciously supported with their presentations, detail technical information and demo products.*

- *Most importantly, I am grateful for CLASSE-Cornell's supports, and for NSF's continuous support in our researches.*

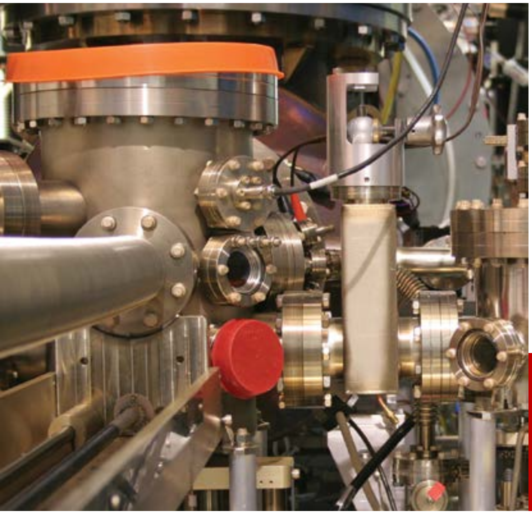


Table of Contents

Schedule & Grading

Hours: Morning sessions: 9:00am – noon (with a brief break)
Afternoon Sessions: 1:30pm – 5:00pm (with a brief break)
Homework office hour: 7:30pm – 9:30pm (in classroom)
Lunch on your own noon – 1:30pm (Class lunch Monday @Mary's Pizza Shack)
Vacuum Instrument demo: Wednesday pm (MKS); Friday AM (Agilent)
Class closes on at noon on Friday

- Monday – Fundamental (Session 1), Gas Sources (Session 2) and Instruments (Session 3)
- Tuesday – Vacuum Pumps (Session 4) and Vacuum Material (Session 5)
- Wednesday – Vacuum Hardware and Fabrications (Session 5)
- Thursday – System Engineering and Vacuum Calculations (1D and MolFlow demonstrations) (Session 6.1) & System Integration (Session 6.2)
- Friday – Beam Vacuum Interactions (Session 7)

- *There are daily homework, and all must submitted by Friday (if not earlier), for grade students*
- *There will be no final exam. Grade will be based on class participation (70%) and homework (30%)*

SESSION 1: VACUUM FUNDEMENTALS

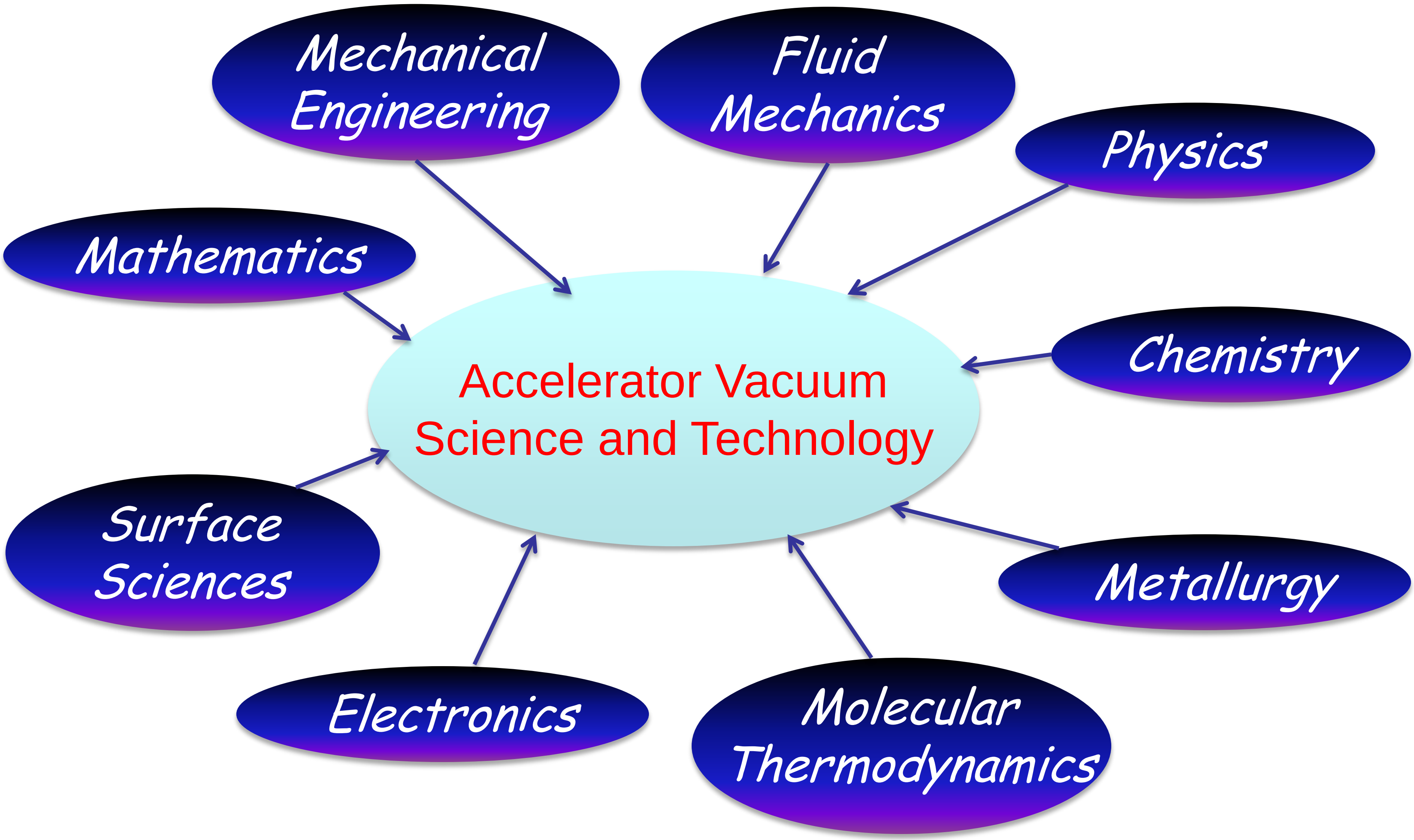
- Vacuum definition and scales
- Gas properties and laws
- Gas flow and conductance

The Basics

What is a Vacuum ?

- *A vacuum is the state of a gas where the density of the particles is lower than atmospheric pressure at the earth's surface*
- *Vacuum science studies behavior of rarefied gases, interactions between gas and solid surfaces (adsorption and desorption), etc.*
- *Vacuum technology covers wide range of vacuum pumping, instrumentations, material engineering, and surface engineering*

Accelerator Vacuum – *highly interdisciplinary*

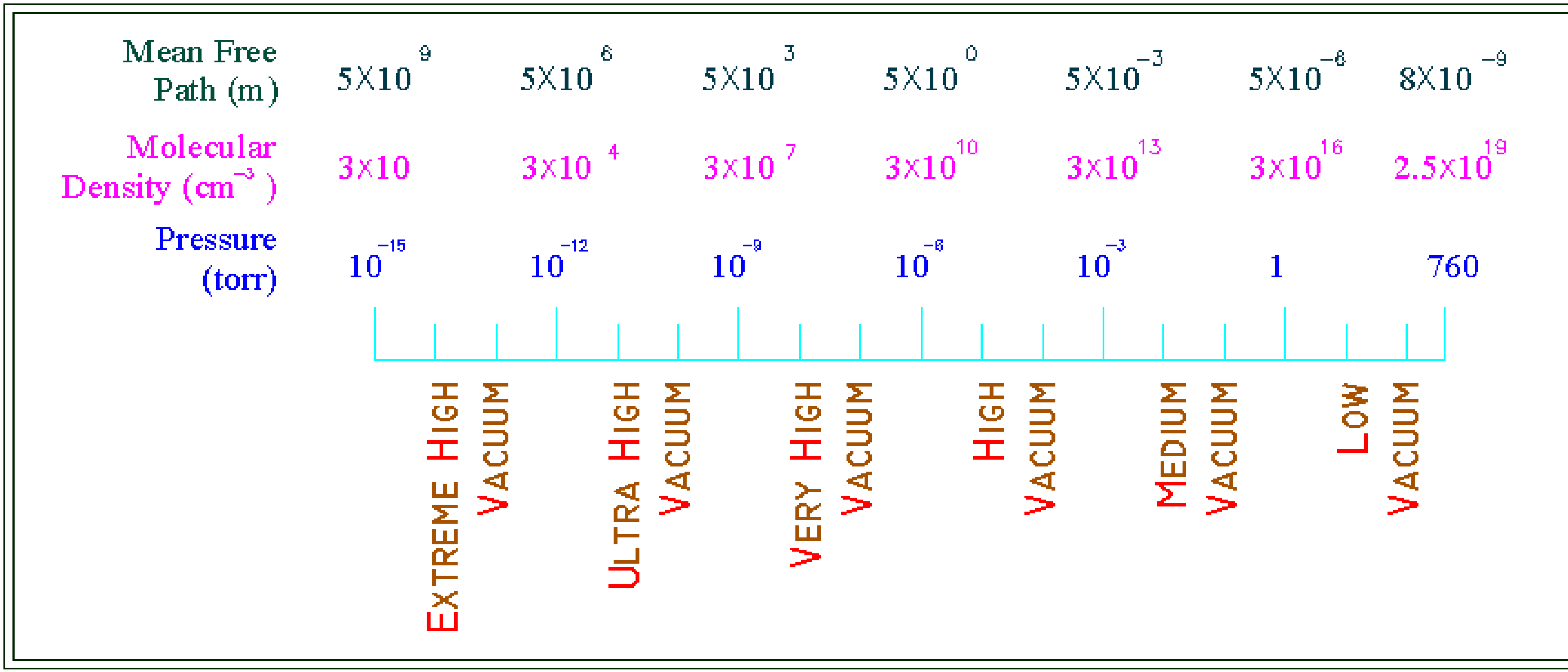


Units, Scales & Molecular Densities

SI Pressure Unit: Pascal (Pa) = N/m^2

Other commonly used units: Torr (mmHg) = 133.3 Pa

mbar = 100 Pa = 0.75 Torr



Example Vacuum Applications

Vacuum range	Applications
Extreme High Vacuum ($P < 10^{-11}$ torr)	*Photo-cathode electron sources (Cornell, JLab,), High intensity ion accelerators, etc.*
Ultra-High Vacuum (10^{-9} to 10^{-11} torr)	*Storage rings, surface sciences, etc.*
High Vacuum (10^{-3} to 10^{-9} torr)	*Device fabrications, medical accelerators, LINACs, mass spectrometry, SEM, etc.*
Medium & Low Vacuum	*Cryo insolation vacuum, coating, vacuum furnaces, beam welders, etc.*

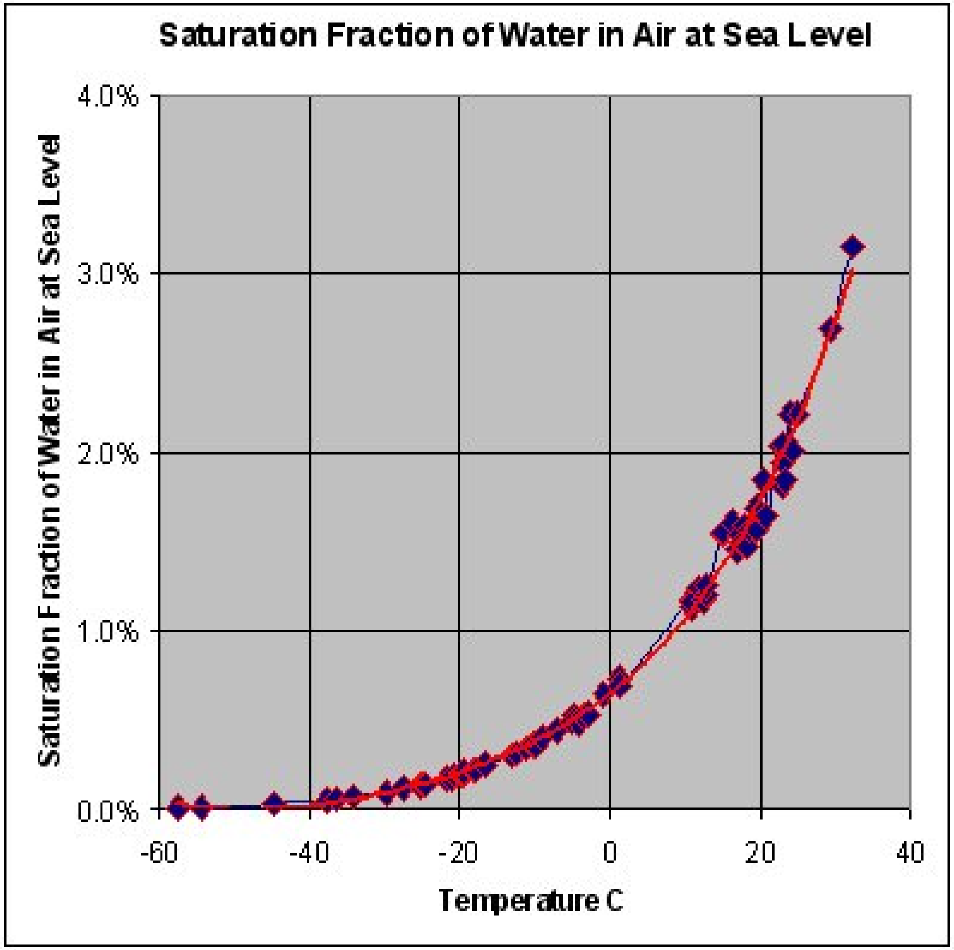
Dry Air Composition and Molecular Mass

Constituent	Volume Content		Molecular Mass	Molecular Mass in Air
	Percent	ppm		
N_2	78.08		28.02	21.88
O_2	20.95		32.00	6.704
Ar	0.934		39.94	0.373
CO_2	0.037		44.01	0.013
H_2		0.5	2.02	---
Ne		18.2	20.18	---
He		5.24	4.00	---
Kr		1.14	83.8	---
Xe		0.087	131.3	---
CH_4		2.0	16.04	---
N_2O		0.5	44.01	---
Total				28.97

Water Vapor in Air

- Water has most detrimental effects in a clean vacuum system
- Water molecular mass:
  18.08 kg/mole
  ('humid'-air is less dense!)
- Water content in air is usually expressed in relative humidity (ϕ):

$$\emptyset = \frac{PP_w}{PP_{w\_sat}} \text{ x } 100\%$$



Gas Properties & Laws

Kinetic Picture of a Ideal Gas

1. The volume of gas under consideration contains a large number of molecules and atoms – statistic distribution applies.
2. Molecules are far apart in space, comparing to their individual diameters.
3. Molecules exert no force on one another, except when they collide. All collisions are elastic (i.e. no internal excitation).
4. Molecules are in a constant state of motion, in all direction equally. They will travel in straight lines until they collide with a wall, or with one another

Maxwell-Boltzmann Velocity Distribution

$$\frac{dn}{dv} = \frac{2N}{\pi^{1/2}}\left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-\left(\frac{m}{2kT}\right)v^2}$$

v – *velocity of molecules (m/s)*

n – *number of molecules with v between v and $v + dv$*

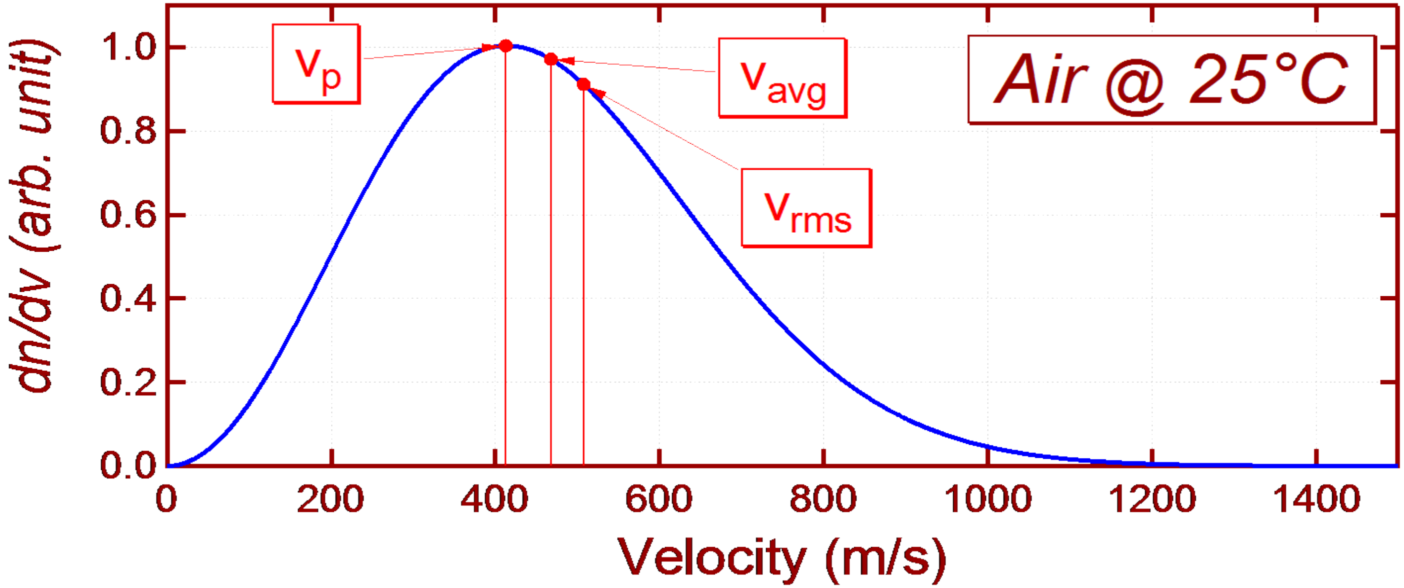
N – *the total number of molecules*

m – *mass of molecules (kg)*

k – *Boltzmann constant, 1.3806503×10^{-23} m^2 kg s^{-2} K^{-1}*

T – *temperature (kelvin)*

A Close Look at Velocity Distribution



- Most probable velocity (m/s): $v_p = \sqrt{2kT/m} = 128.44\sqrt{T/M_{mole}}$
- Arithmetic mean velocity: $v_{avg} = \sqrt{8kT/\pi m} = 1.128 v_p$
- Root Mean Squared velocity: $v_{rms} = \sqrt{3kT/m} = 1.225 v_p$

Velocity depends on mass and temperature, but independent of pressure

Speed of Sound in 'Ideal' Gases

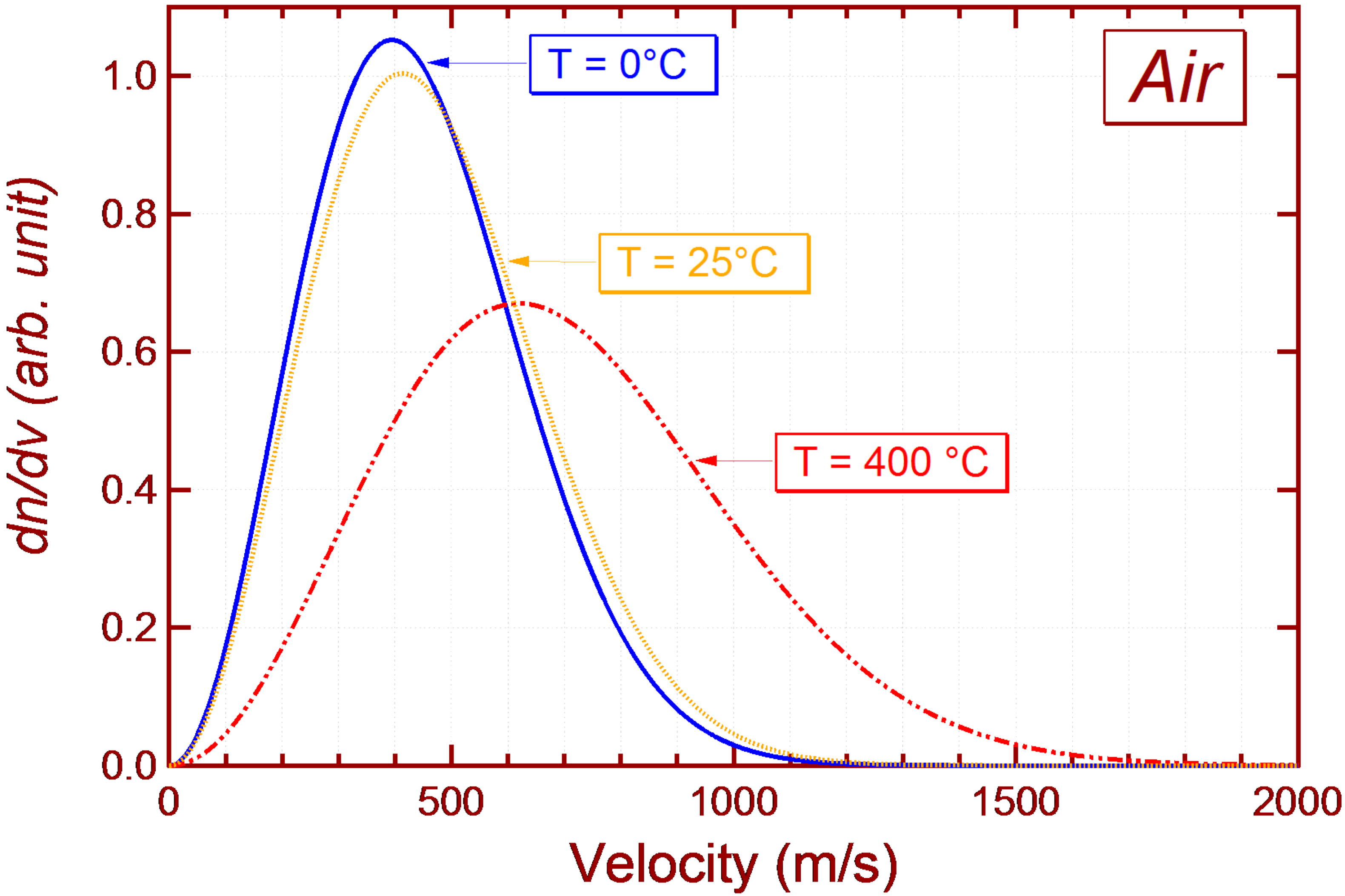
$$v_{sound} = \sqrt{\gamma \frac{P}{\rho}} = \sqrt{\frac{\gamma kT}{m}}$$ *where* $\gamma = C_p / C_V$ *is the isentropic expansion factor*

γ=1.667 (atoms); $\gamma\approx$1.40 (diatomic); $\gamma\approx$1.31 (triatomic)

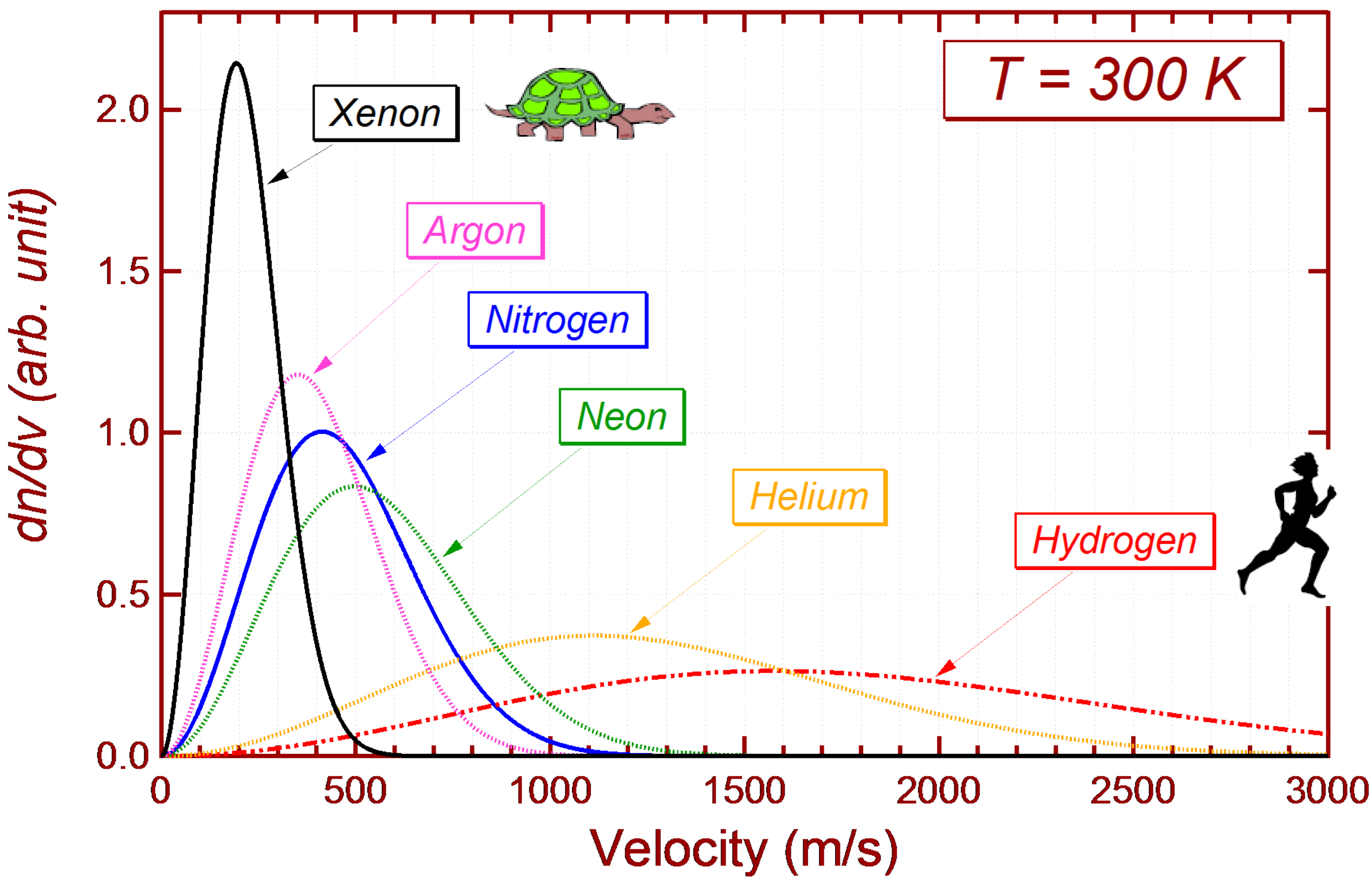
	Mono-atomic	**Di-atomic**	**Tri-atomic**
v_p/v_{sound}	1.10	1.19	1.24
v_{avg}/v_{sound}	1.24	1.34	1.39
v_{rms}/v_{sound}	1.34	1.46	1.51

Though not directly related, the characteristic gas speeds are close to the speed of sound in ideal gases.

Molecules Moves Fast at Higher Temperatures



Light Molecules Moves Faster



Maxwell-Boltzmann Energy Distribution

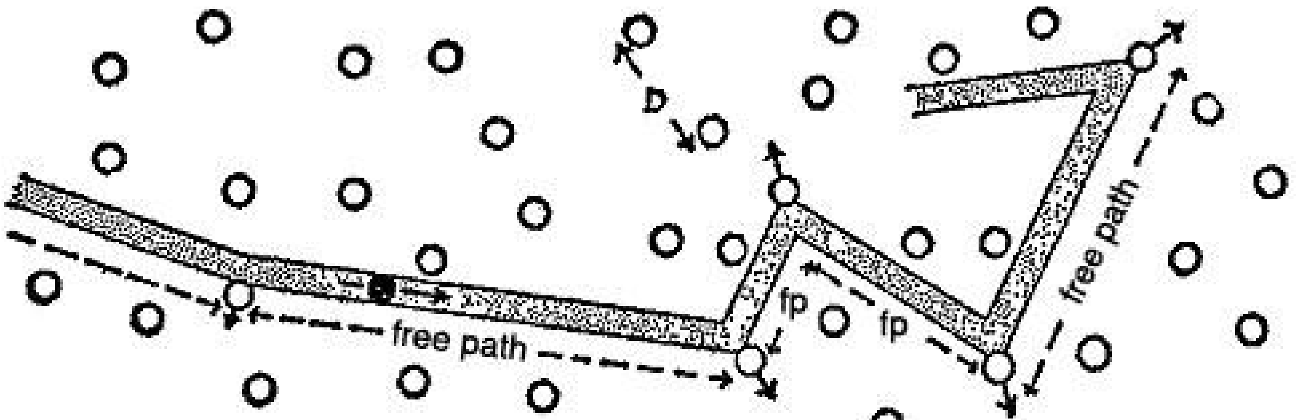
$$\frac{dn}{dE} = \frac{2N}{\pi^{1/2}} \frac{E^{1/2}}{(kT)^{3/2}} e^{-\left(\frac{E}{2kT}\right)}$$

- Average Energy: $E_{avg} = 3kT/2$
- Most probable Energy: $E_p = kT/2$

Neither the energy distribution nor the average energy of the gas is a function of the molecular mass. They ONLY depend on temperature !

Mean Free Path

The mean free path is the average distance that a gas molecule can travel before colliding with another gas molecule.



Mean free path of a gas molecule

Mean Free Path is determined by:

size of molecules, density (thus pressure and temperature)

Mean Free Path Equation

$$\lambda = \frac{1}{\sqrt{2}\pi d_0^2 n} = \frac{kT}{\sqrt{2}\pi d_0^2 P}$$

λ - *mean free path (m)*

d_0 - *diameter of molecule (m)*

n - *Molecular density (m^{-3})*

T - *Temperature (Kevin)*

P - *Pressure (Pascal)*

k - *Boltzmann constant,* 1.38×10^{-23} m^2 kg s^{-2} K^{-1}

Mean Free Path – Air @ 22℃

$$\lambda(cm) = \frac{0.67}{P(Pa)} = \frac{0.005}{P(Torr)}$$

P (torr)	760	1	10^{-3}	10^{-6}	10^{-9}
λ (cm)	6.6×10^{-6}	5.1×10^{-3}	5.1	5100	5.1×10^{6}

For air, average molecular diameter = $3.74\ \times10^{-8}$ cm

Properties of Some Gases at R.T.

Properties	H_2	He	CH_4	Air	O_2	Ar	CO_2
v_{avg} (m/s)	1776	1256	628	467	444	397	379
d_0 (10^{-10} m)	2.75	2.18	4.19	3.74	3.64	3.67	4.65
λ_{atm} (10^{-8} m)	12.2	19.4	5.24	6.58	6.94	6.83	4.25
ω_{atm} (10^{10} s^{-1})	1.46	0.65	1.20	0.71	0.64	0.58	0.89

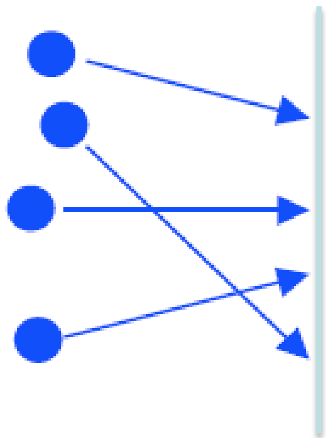
→ v_{avg} **: mean velocity**
→ d_0 **: molecular/atomic diameter**
→ λ_{atm} **: mean free length at atmosphere pressure**
→ ω_{atm} **: mean collision rate,** $\omega_{atm} = v_{avg} / \lambda_{atm}$

Particle Flux

Particle Flux *– rate of gas striking a surface or an imaginary plane of unit area*

Kinetic theory shows the flux as:

$$\Gamma(m^{-2}\cdot s^{-1}) = \frac{1}{4} n v_{avg} = n\sqrt{\frac{kT}{2\pi m}}$$



where n is the gas density

Particle flux is helpful in understanding gas flow, pumping, adsorption and desorption processes.

Particle Flux – Examples

P	n	Particle Flux ($m^{-2}\cdot s^{-1}$)				
(torr)	m^{-3}	H_2	H_2O	CO/N_2	CO_2	Kr
760	2.5×10^{25}	1.1×10^{28}	3.6×10^{27}	2.9×10^{27}	2.3×10^{27}	1.7×10^{27}
10^{-6}	3.2×10^{16}	1.4×10^{19}	4.8×10^{18}	3.8×10^{18}	3.1×10^{18}	2.2×10^{18}
10^{-9}	3.2×10^{13}	1.4×10^{16}	4.8×10^{15}	3.8×10^{15}	3.1×10^{15}	2.2×10^{15}

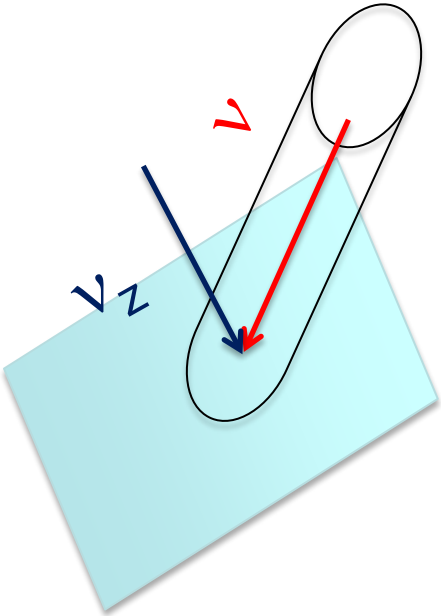
Typical atomic density on a solid surface: $5.0\sim12.0\times10^{18}$ m^{-2}
Thus monolayer formation time at 10^{-6} torr: ~ 1-sec !

A commonly used exposure unit: 1 Langmuir = 10^{-6} torr·sec

The Ideal Gas Law

→ ν molecules striking a unit area, with projected velocity of ν_z exert an impulse, or pressure, of:

$$P = 2m \sum v_z \cdot v$$



→ Integrate over all velocity, based on kinetics,

$$P = \tfrac{1}{3} nm v_{rms}^2 \qquad \text{(A)}$$

→ The average kinetic energy of a gas is:

$$E = \tfrac{1}{2} m v_{rms}^2 = \tfrac{3}{2} kT \qquad \text{(B)}$$

→ Equations (A) and (B) results in ***the Ideal Gas Law***:

$$P = nkT$$

Gas Laws

Charles' Law

Volume vs. temperature

Boyle's Law

Pressure vs. Volume

Avogadro's Law

Volume vs. Number of molecules

All these laws derivable from the Ideal Gas Law

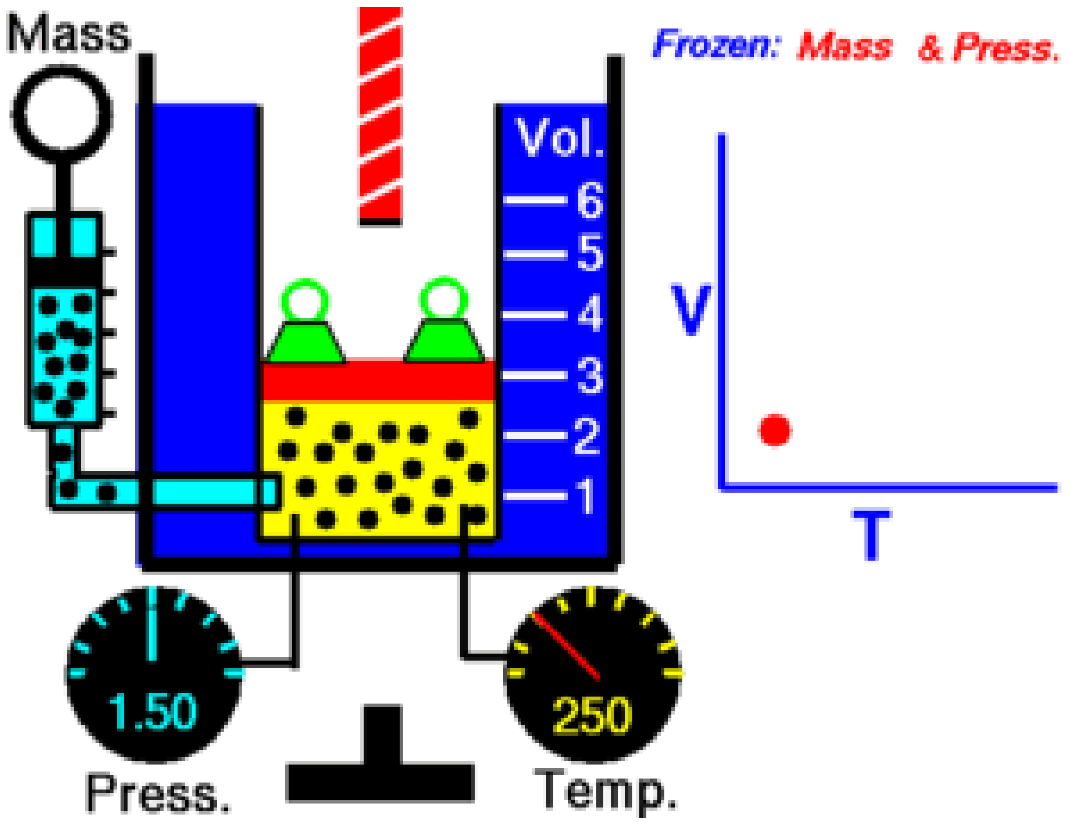
They apply to all molecules and atoms, regardless of their sizes

Charles' Law

The volume of a fixed amount of gas at a fixed pressure will vary proportionally with absolute temperature.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

(N, P Constant)

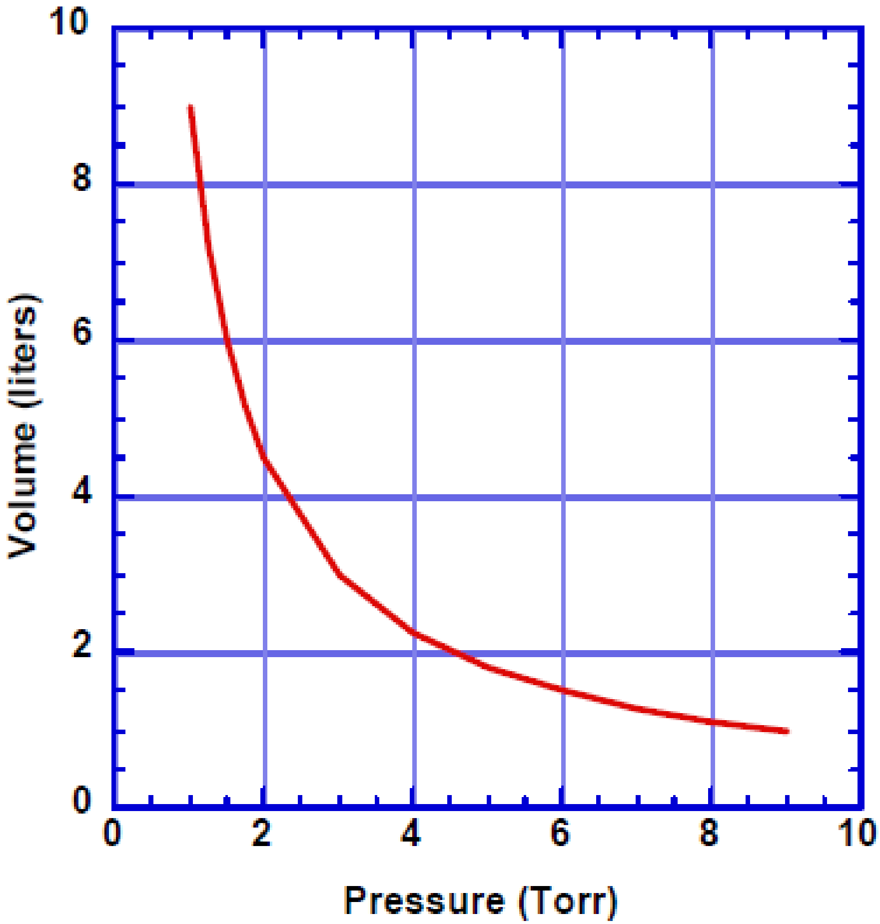


Boyle's Law

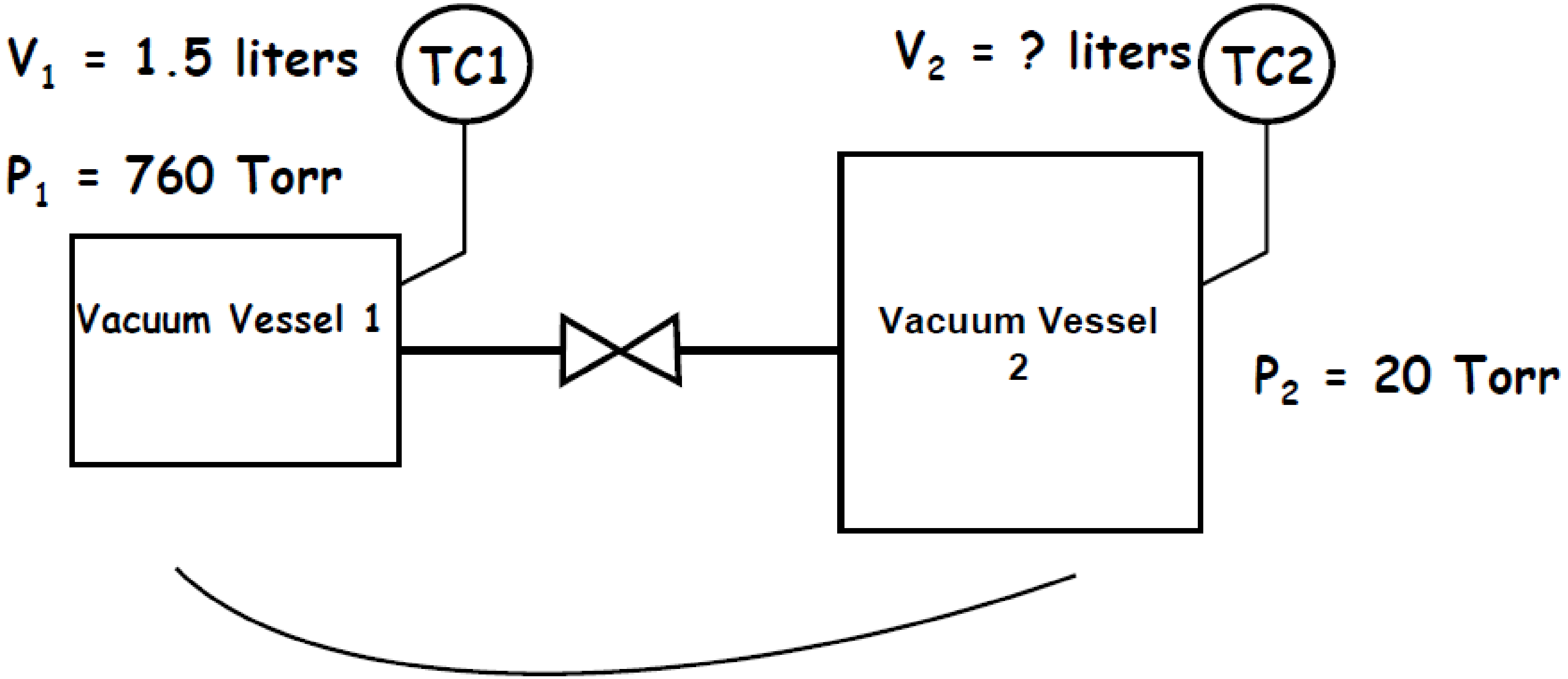
For a fixed amount of gas at a fixed temperature, its pressure is inversely proportional to its volume.

$$P_1 V_1 = P_2 V_2$$

(N, T Constant)



Finding volume of a vessel with Boyle's Law



Boyle's Law can also be used to expend gauge calibration range via expansion

Avogadro's Law

Equal volumes of gases at the same temperature and pressure contain the same number of molecules regardless of their chemical nature and physical properties

$$\frac{V_1}{N_1} = \frac{V_2}{N_2}$$

(P, T Constant)

Avogadro's Number

At **S**tandard **T**emperature and **P**ressure (STP, 0°C at 1 atmosphere), a mole of ideal gas has a volume of V_o, and contains N_A molecules and atoms:

$$N_A = 6.02214 \times 10^{23}\ mol^{-1}$$

$$V_o = 22.414\ L/mol$$

Definition of mole: (to be obsolete in new SI system!!)
the number of atoms of exact 12-gram of Carbon-12

Among other changes in the new SI, N_A will be set to be a exact value as shown above, and mole will be redefined.

Partial Pressure – The Dalton's Law

The total pressure exerted by the mixture of non-reactive gases equals to the sum of the partial pressures of individual gases

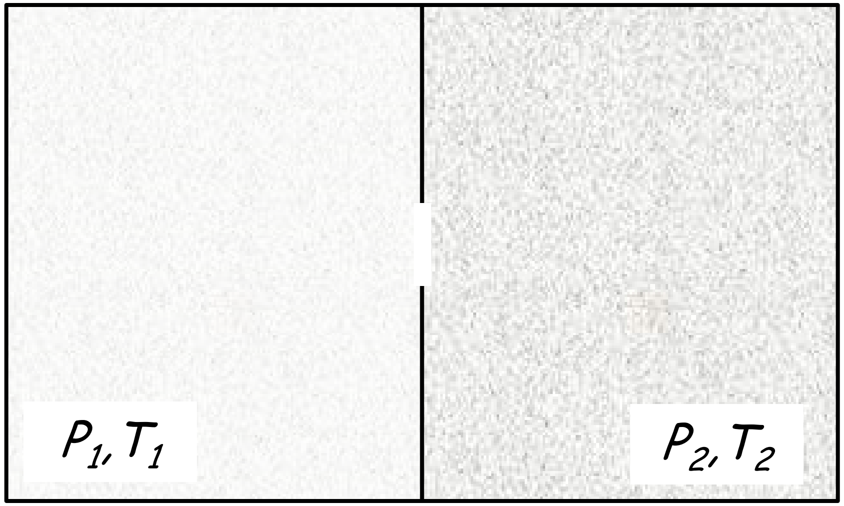
$$P_{total} = p_1 + p_2 + \ldots + p_n$$

where p_1, p_2, …, p_n represent the partial pressure of each component.

Applying the Ideal Gas Law:

$$P_{total} = (n_1 + n_2 + \ldots + n_n)kT$$

Thermal Transpiration



At molecular flow (λ >> d), gas flux through an orifice:

$$\Gamma_{1,2} = \frac{n_{1,2}}{4}\left(\frac{8kT_{1,2}}{\pi m}\right)^{1/2} = \frac{P_{1,2}}{\left(2\pi k m T_{1,2}\right)^{1/2}}$$

- *Temperature gradient causes mass flow even without pressure gradient (thermal diffusion);*
- *OR in a steady state (zero net flow), a pressure difference exists between different temperature zones.*

$$\frac{P_1}{P_2} = \sqrt{\frac{T_1}{T_2}}$$

This can be used to infer the pressures within furnaces or cryogenic enclosures with a vacuum gauge outside the hot/cold zone.

Graham's Law of Gas Diffusion

Considering a gas mixture of two species, with molar masses of M_1 and M_2. The gases will have same kinetic energy:

$$\tfrac{1}{2} M_1 u_1^2 = \tfrac{1}{2} M_2 u_2^2 \quad \Rightarrow \quad \frac{u_1}{u_2} = \sqrt{\frac{M_2}{M_1}}$$

Assuming diffusion rate is proportional to the gas mean velocity, then rate of diffusion rates:

$$\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$$

Graham's law was the basis for separating ^{235}U and ^{238}U by diffusion of $^{235}UF_6$ and $^{238}UF_6$ gases in the Manhattan Project.

Elementary Gas Transport Phenomena

→ *Viscosity – molecular momentum transfer, depending on density, velocity, mass and mean free path*

$$\eta = 0.499 nm v\lambda \qquad \text{(Pa-s in SI)}$$

→ *Thermal Conductivity – molecular energy transfer*

$$K = \frac{1}{4}(9\gamma - 5)\eta c_v \qquad \text{(W/m-K in SI)}$$

where $\gamma = c_p/c_v$, c_p, and c_v are specific heat in a constant pressure and a constant volume process, respectively.

γ=1.667 (atoms); $\gamma \approx$1.40 (diatomic); $\gamma \approx$1.31 (triatomic)

No longer ideal gas, as internal energy of a molecule is in play

Gas Flows

Gas Flow Regimes – Knudsen Number

The flow of gases in a vacuum system is divided into three regimes. These regimes are defined by specific value ranges of a dimensionless parameter, known as the **Knudsen Number, K_n**

$$K_n = \frac{\lambda}{a}$$

λ ← Mean free path

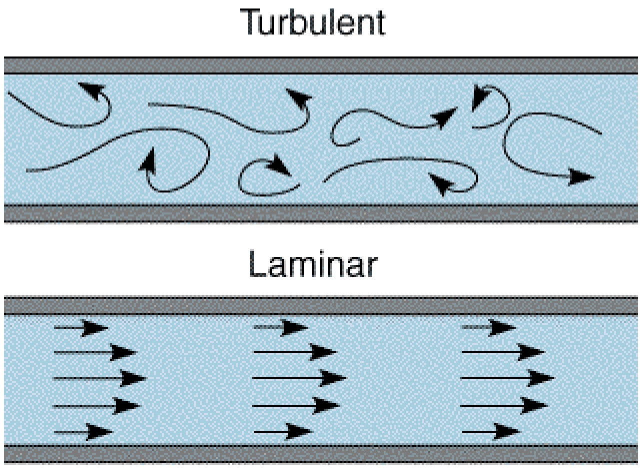
a ← Characteristic dimension of flow channel (for example, a pipe diameter)

Three Gas Flow Regimes

→ Viscous Flow:

Characterized by molecule-molecule collisions

$$K_n = \frac{\lambda}{a} < 0.01$$

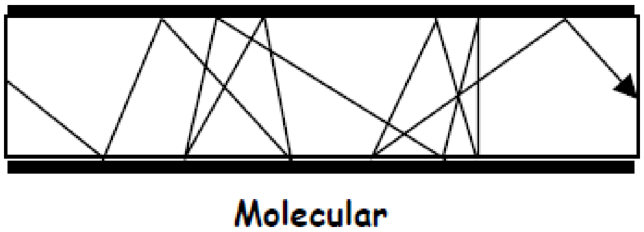


→ Transition Flow: $0.01 < K_n < 1.0$

→ Molecular Flow:

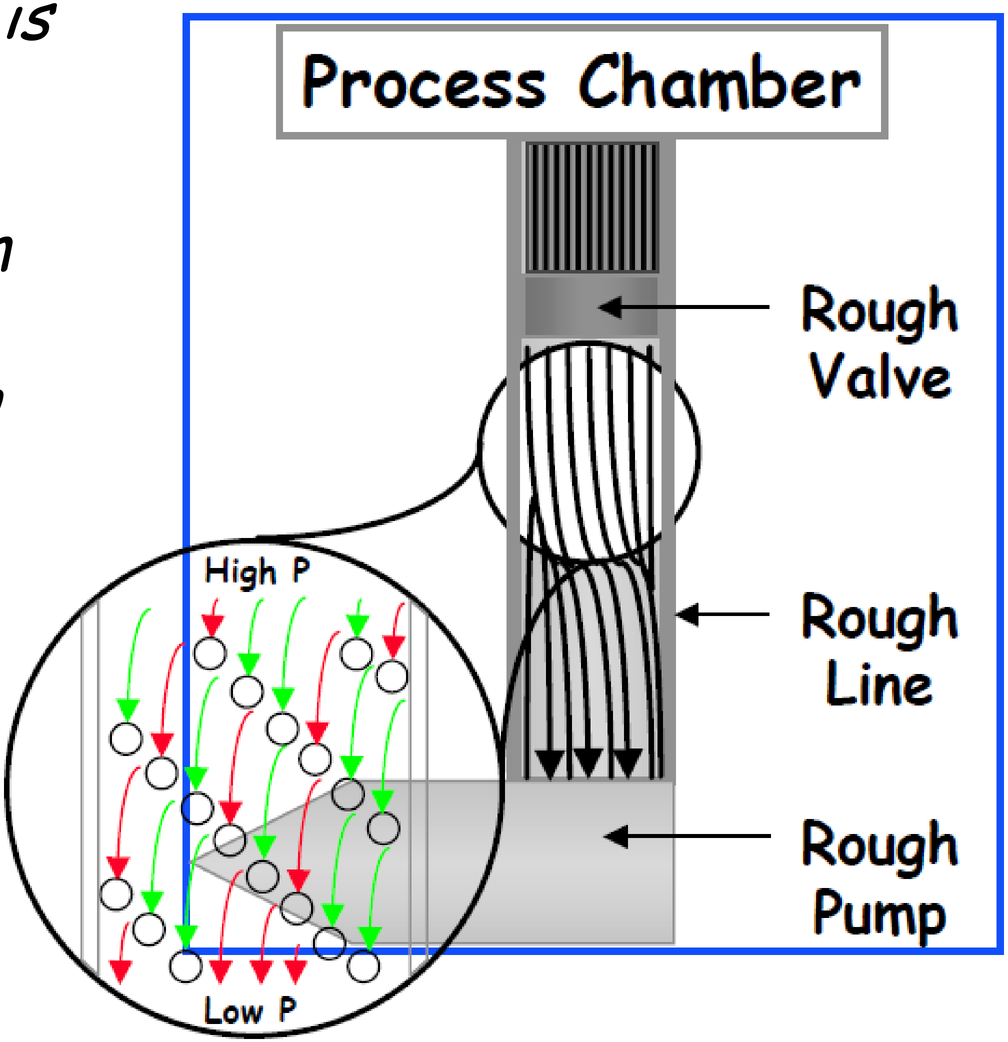
Characterized by molecule-wall collisions

$$K_n > 1.0$$



Viscous (or Continuum) Flow

- → *The character of the flow is determined by gas-gas collisions*
- → *Molecules travel in uniform motion toward lower pressure, molecular motion 'against' flow direction unlikely*
- → *The flow can be either turbulent or laminar, characterized by another dimensionless number, the Reynolds' number, R*



Viscous Flow – Reynolds' Number

Reynolds' Number, Re is the ratio of inertia vs. viscosity

$$\mathrm{Re} = \frac{U\rho d}{\eta}$$

U – stream velocity
ρ – gas density
d – pipe diameter
η – viscosity

→ *Laminar Flow: zero flow velocity at wall*

$R < 1200$

→ *Turbulent Flow: eddies at wall and strong mixing*

$R > 2100$

Throughput

Throughput is defined as the quantity of gas flow rate, that is, the volume of gas at a known pressure passing a plane in a known time

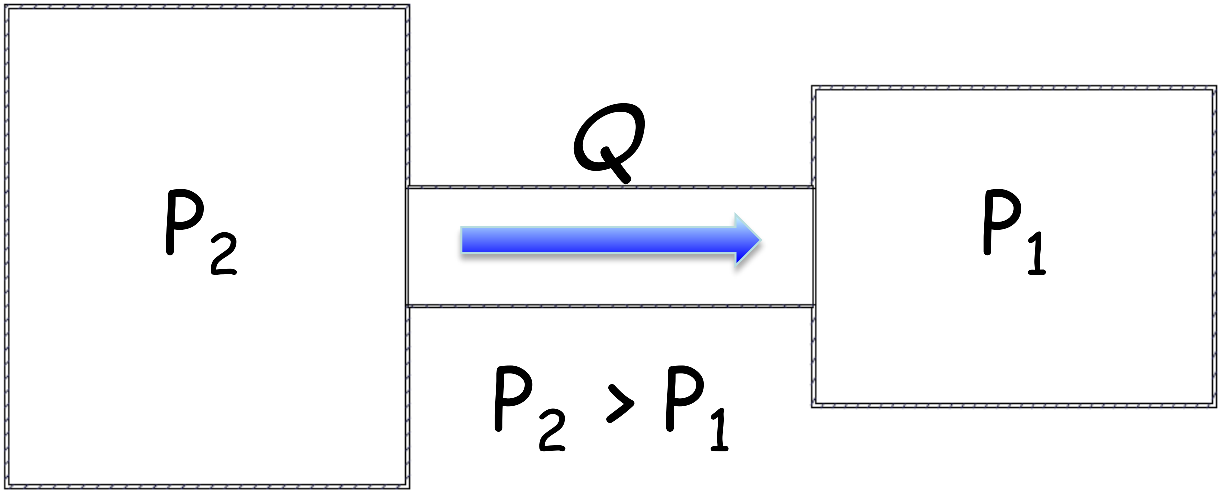
$$Q = \frac{d(PV)}{dt}$$

In SI unit, [Q] = Pa-m^3/s (= 7.5 torr-liter/s)

Interesting Point:

Pa-m^3/s = N-m/s = J/s = Watt !

Gas Conductance – Definition



→ *The flow of gas in a duct or pipe depends on the pressure differential, as well on the connection geometry*

→ *The gas conductance of the connection is defined:*

$$C = \frac{Q}{P_2 - P_1}$$

In SI unit: m^3/s

Commonly used: liter/s

1-m^3/s = 1000-liter/s

Vacuum Pump – Pumping Speed

- *Defined as a measure of volumetric displacement rate, (m^3/sec, cu-ft/min, liter/sec, etc.)*

$$S = {dV}/{dt}$$

- *The throughput of a vacuum pump is related the pressure at the pump inlet as:*

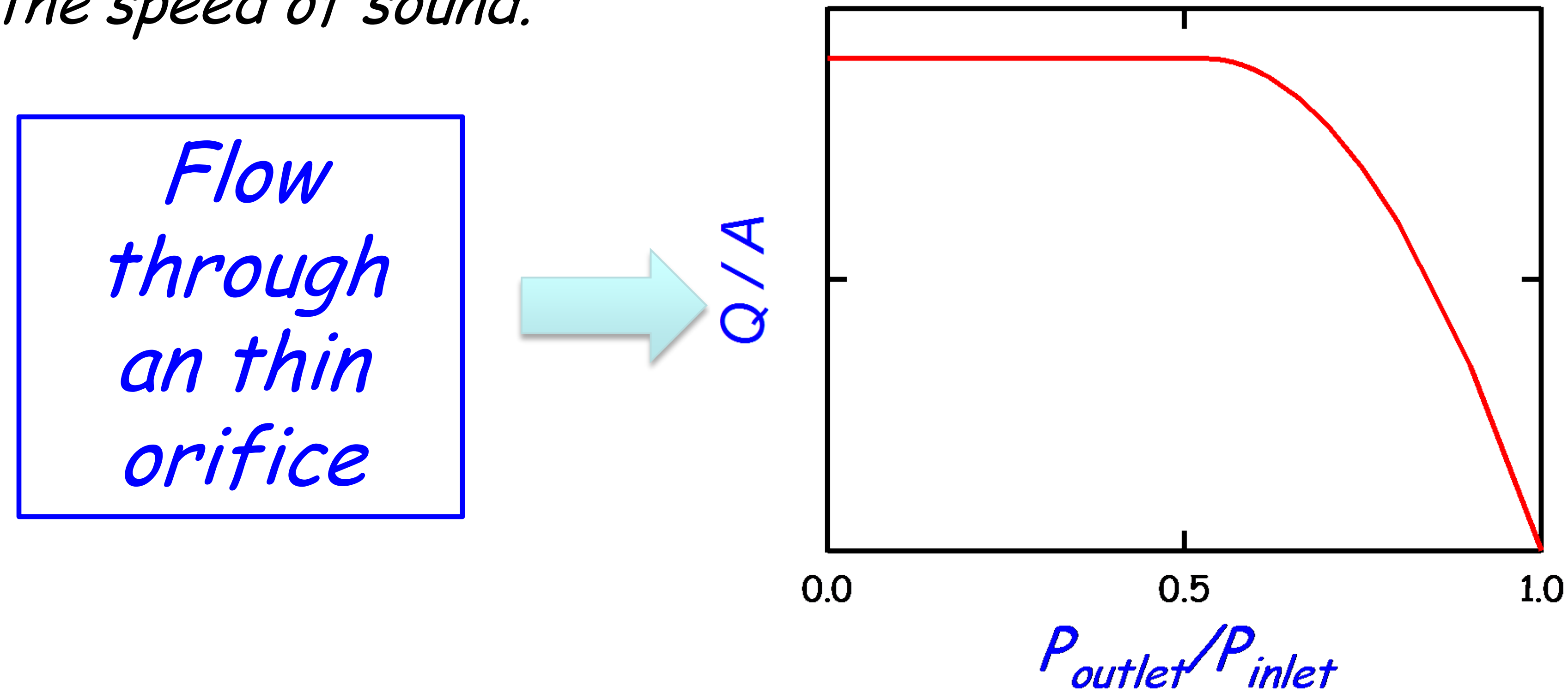
$$Q = P \cdot \frac{dV}{dt} = P \cdot S$$

Gas Conductance – Continuum Flow

- Viscous flow is usually encountered during vacuum system roughing down processes.
- Gas flow in the viscous flow regime is very complicated. A great amount of theoretic works can be found in the literatures on the subject.
- Direct simulation Monte Carlo (DSMC) codes are widely used to calculate the flow in the region.
- Depending the value of Reynolds' number, the flow can be either laminar or turbulent.
- Turbulent flow is usually to be avoided (by 'throttling'), to reduce contamination (to the upstream system), and to reduce vibration.

Gas Conductance – Continuum Flow (2)

- The gas throughput (and conductance) is dependent on both the 'inlet' pressure and the 'outlet' pressure in most situations, and (of cause) on the piping geometry.
- The flow usually increases with reduced outlet pressure, but may be "choked" when the gas stream speed exceeding the speed of sound.



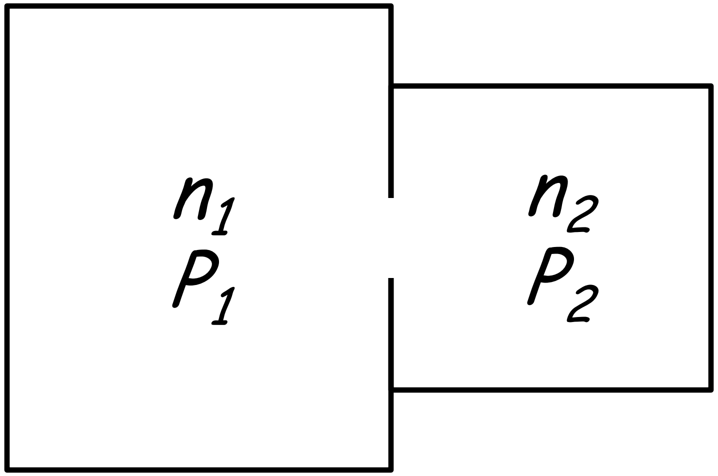
Molecular Flow – Orifices

If two large vessels are connected by an thin orifice of opening area A, then gas flow from vessel 1 to vessel 2 is given by the particle flux exchange:

$$Q = \frac{kT}{4} vA(n_1 - n_2) = \frac{v}{4} A(P_1 - P_2)$$

From definition of the conductance:

$$C = \frac{Q}{P_1 - P_2} = \frac{v}{4} A$$



Molecular Flow – Orifices (2)

With: $$v = \sqrt{8kT/\pi m}$$

We have: $$C_{Orifice}(m^3/s) = 36.24\sqrt{\frac{T}{M_{amu}}}A(m^2)$$

where: T is temperature in Kevin; M_{amu} is molecular mass in atomic mass unit

For air (M_{amu}=28.97) at 22°C:

$$C\,(m^3/s) = 116\,A\,(m^2)$$

Or: $$C\,(L/s) = 11.6\,A\,(cm^2)$$

Molecular Flow – Long Round Tube

From Knudson (1909)

$$C = \frac{\pi}{12} v \frac{d^3}{l} = 37.94 \sqrt{\frac{T}{M_{amu}}} \frac{d^3}{l}$$

where T is the temperature in Kevin, M the molecular mass in AMU, d and l are diameter and length of the tube in meter, with **l >> d**

For air (M_{amu}=28.97) at 22°C:

$$C(m^3/s) = 121 \frac{d^3}{l}$$

Molecular Flow – Short Tube

No analytical formula for the short pipe conductance. But for a pipe of constant cross section, it is common to introduce a parameter called transmission probability, α, so that:

$$C = \alpha C_{Orifice} = \alpha \frac{v}{4} A$$

where A is the area of the pipe cross section.

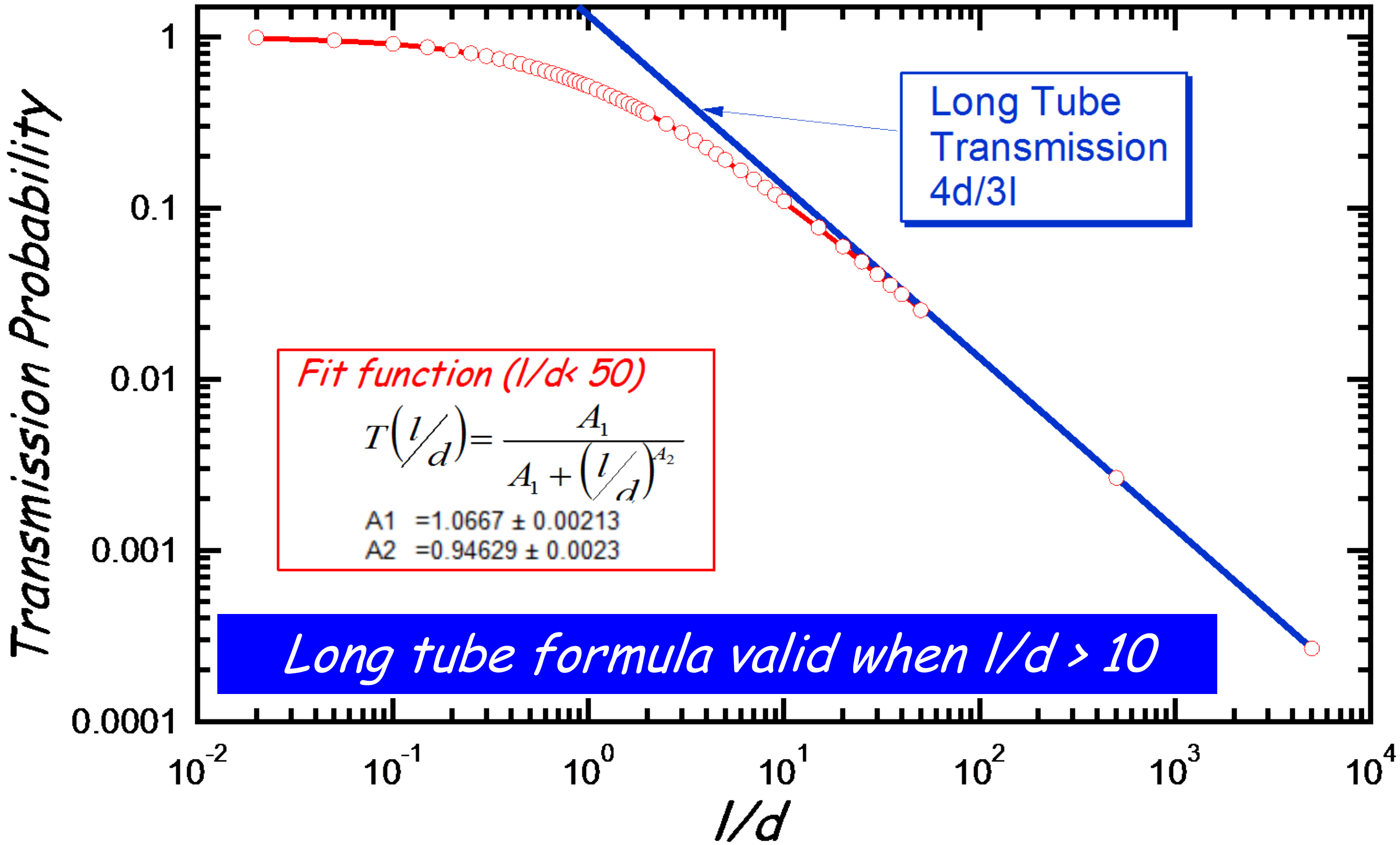
For long round tube, the transmission probability is:

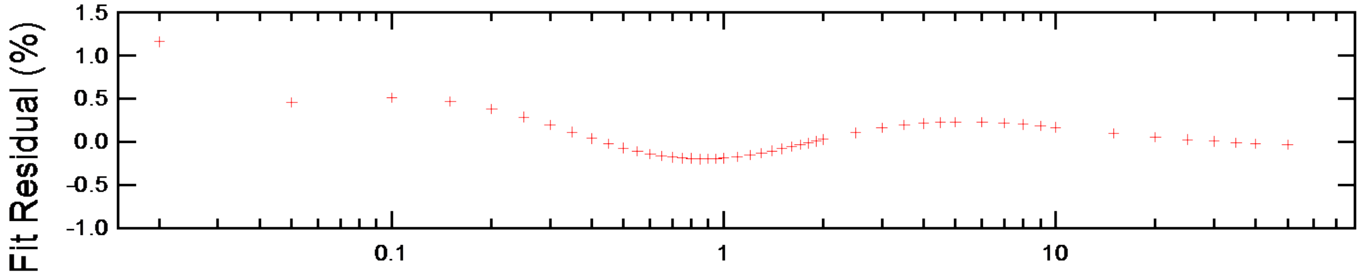
$$\alpha_{long\_tube} = {4d}/{3l}$$

Molecular Flow – Transmission of Round Tube

l/d	a	l/d	a	l/d	a
0.00	1.00000	0.9	0.53898	5.0	0.19099
0.05	0.95240	1.0	0.51423	6.0	0.16596
0.10	0.90922	1.1	0.49185	7.0	0.14684
0.15	0.86993	1.2	0.47149	8.0	0.13175
0.20	0.83408	1.3	0.45289	9.0	0.11951
0.25	0.80127	1.4	0.43581	10	0.10938
0.30	0.77115	1.5	0.42006	15	0.07699
0.35	0.74341	1.6	0.40548	20	0.05949
0.40	0.71779	1.8	0.37935	25	0.04851
0.45	0.69404	2.0	0.35658	30	0.04097
0.50	0.69178	2.5	0.31054	35	0.03546
0.55	0.65143	3.0	0.27546	40	0.03127
0.60	0.63223	3.5	0.24776	50	0.02529
0.70	0.59737	4.0	0.22530	500	2.65×10^{-2}
0.80	0.56655	4.5	0.20669	5000	2.66×10^{-3}

Molecular Flow – Transmission of Round Tube

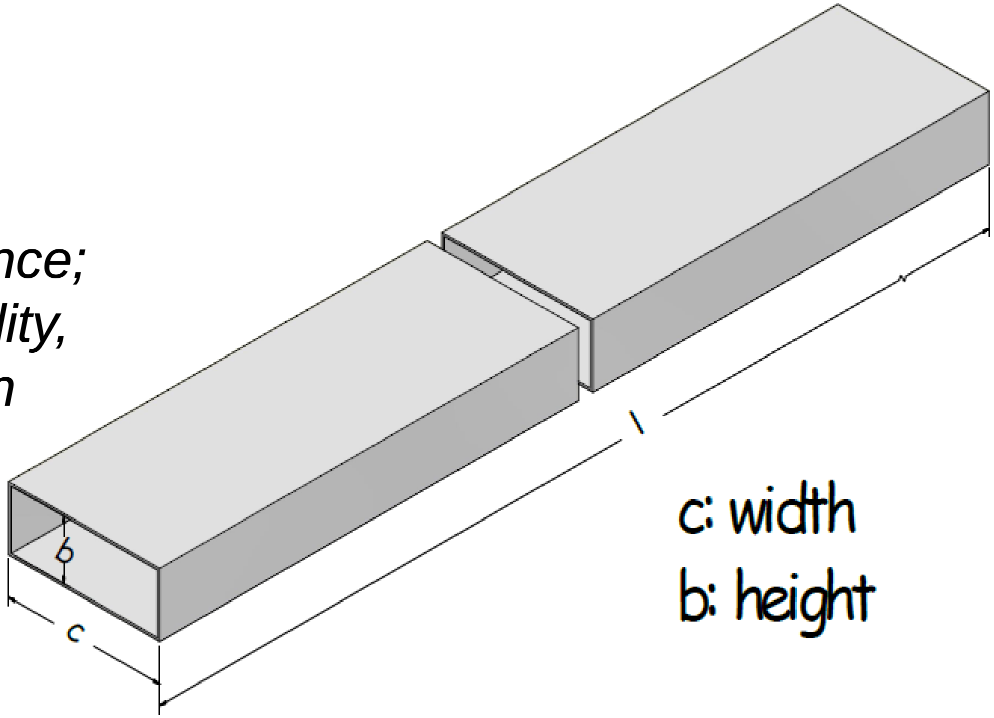




Gas Conductance of Rectangular Ducts

$$C = \alpha_{rect}(b,c,l)\cdot C_0$$

C_0 *is the entrance aperture conductance;*
$\alpha_{rect}(b,c,l)$ *is the transmission probability, depending on geometry and length of the duct*



$$C_0 = 3.64bc(T/M)^{1/2}$$

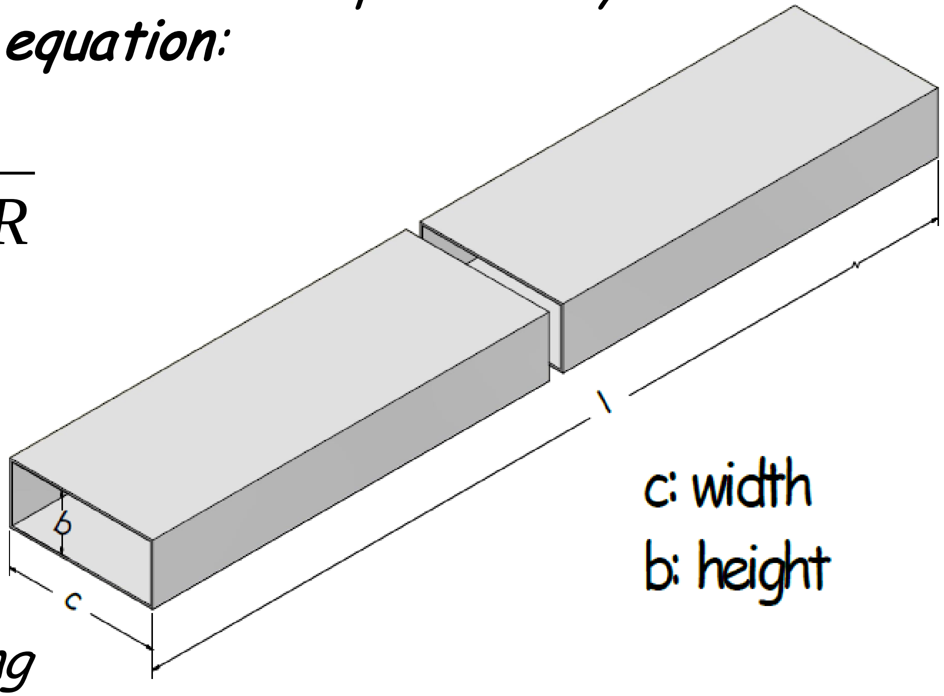
C_0 *in liter/sec; b, c are height and width of the duct in cm*
T is gas temperature in Kevin; M is the molar mass of the gas

REF. ***D.J. Santeler and M.D. Boeckmann, J. Vac. Sci. Technol. A9 (4) p.2378, 1991***

Transmission of Long Rectangular Ducts

For a very long rectangular duct, the transmission probability can be calculated by the modified **Knudson equation**:

$$\alpha_{vl} = \frac{8bcK}{3l(b+c)} = \frac{1}{3L(1+R)/8KR}$$



where **L** and **R** are defined as:

$$L = l/b \qquad R = c/b \, (c > b)$$

and K is a correcting factor reflecting geometrical shape of the duct

R	1.0	1.5	2.0	3.0	4.0	6.0	8.0	12.0	16.0	24.0
K(R)	1.115	1.127	1.149	1.199	1.247	1.330	1.398	1.505	1.588	1.713

REF. D.J. Santeler and M.D. Boeckmann, J. Vac. Sci. Technol. A9 (4) p.2378, 1991

Transmission of Rectangular Ducts

Short rectangular duct

$$\alpha_s = \frac{1}{1 + L(1+R)/2R}$$

Long rectangular duct

$$\alpha_l = \frac{1}{1 + 3L(1+R)/8RK}$$

Rectangular duct of any length – empirical equation

$$\alpha = \frac{1 + D(L/R)^E}{(1/\alpha_s) + \left[D(L/R)^E\right]/\alpha_l}$$

where D and E are fitting parameters

Now we need to know the values of the three parameters: K, D & R

Transmission of Rectangular Ducts – Computed

$L = l/b$	$c/b = 1$	$c/b = 1.5$	$c/b = 2$	$c/b = 3$	$c/b = 4$
0.01	0.990 2113	0.991 8139	0.992 6161	0.993 4187	0.993 8202
0.02	0.980 7386	0.983 8524	0.985 4125	0.986 9740	0.987 7552
0.04	0.962 5832	0.968 4966	0.971 4684	0.974 4415	0.975 9297
0.07	0.936 9894	0.946 6600	0.951 5284	0.956 4141	0.958 8612
0.1	0.913 058	0.926 047	0.932 612	0.939 213	0.942 520
0.2	0.842 781	0.864 522	0.875 652	0.886 902	0.892 559
0.4	0.733 42	0.765 85	0.782 92	0.800 43	0.809 31
0.7	0.617 78	0.657 51	0.679 29	0.702 23	0.714 03
1.0	0.5363	0.5786	0.602 62	0.6285	0.642 13
2.0	0.3780	0.4192	0.4444	0.4733	0.4893
4.0	0.2424	0.2759	0.2977	0.3245	0.3404
7.0	0.1596	0.1848	0.2020	0.2242	0.2380
10.0	0.1195	0.1397	0.1537	0.1723	0.1843
20.0	0.0655	0.0776	0.0864	0.0984	0.1066
40.0	0.0346	0.041	0.0464	0.053	0.058
70.0	0.020	0.024	0.0275	0.032	0.035
100.0	0.014	0.017	0.019	0.023	0.025

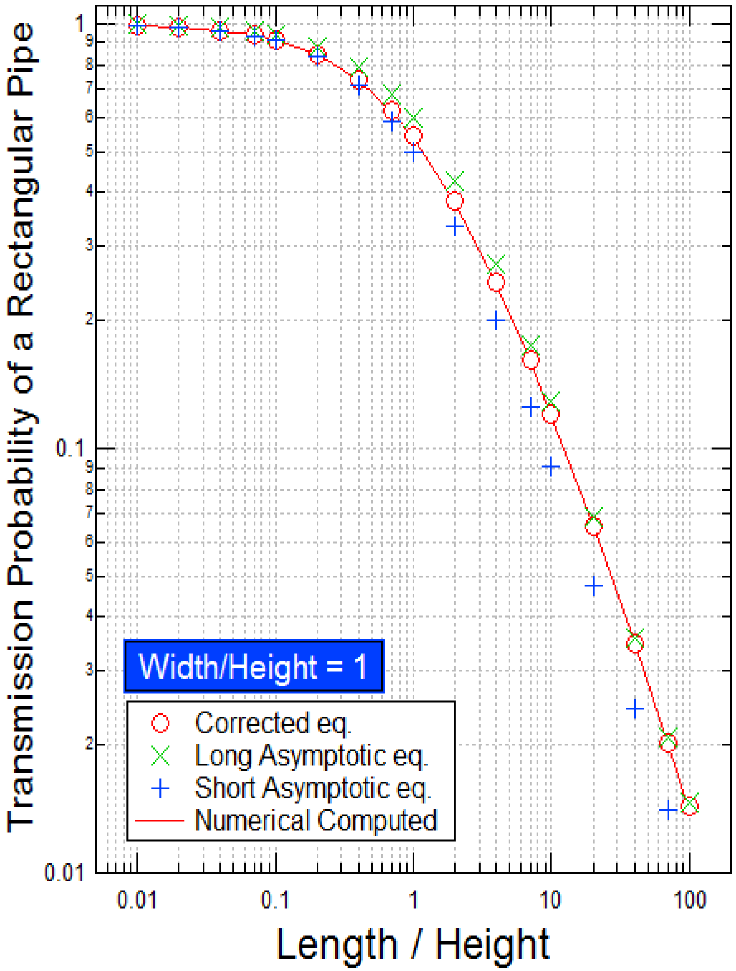
$L = l/b$	$c/b = 6$	$c/b = 8$	$c/b = 12$	$c/b = 16$	$c/b = 24$
0.01	0.994 2217	0.994 4226	0.994 6234	0.994 7238	0.994 8242
0.02	0.988 5400	0.988 9301	0.989 3201	0.989 5152	0.989 7103
0.04	0.977 4186	0.978 1632	0.978 9080	0.979 2803	0.979 6527
0.07	0.961 3102	0.962 5390	0.963 7629	0.963 4749	0.964 9867
0.1	0.945 830	0.947 486	0.949 141	0.949 969	0.950 797
0.2	0.898 233	0.901 070	0.903 908	0.905 327	0.906 745
0.4	0.818 24	0.822 72	0.827 21	0.829 45	0.831 69
0.7	0.726 02	0.732 05	0.738 09	0.741 13	0.744 16
1.0	0.656 04	0.663 08	0.670 16	0.673 71	0.677 27
2.0	0.5063	0.5150	0.5240	0.5285	0.5330
4.0	0.3583	0.3679	0.3781	0.3833	0.3885
7.0	0.2545	0.2639	0.2742	0.2796	0.2852
10.0	0.1991	0.2078	0.2177	0.2230	0.2287
20.0	0.1171	0.1238	0.1319	0.1366	0.1419
40.0	0.0652	0.0695	0.075	0.078	0.083
70.0	0.039	0.042	0.046	0.048	0.052
100.0	0.028	0.030	0.033	0.035	0.038

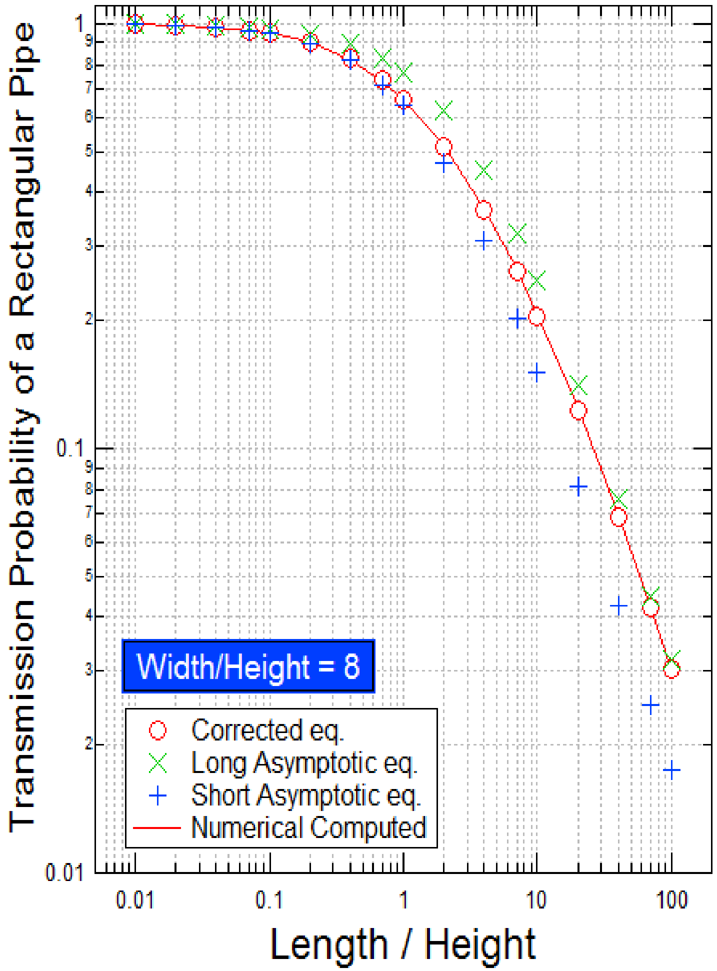
Fitted parameters for transmission equation (for some aspect ratio)

R=c/b	1	1.5	2	3	4	6	8	12	16	24
D	0.8341	0.9446	1.0226	1.1472	1.2597	1.4725	1.6850	2.1012	2.5185	3.3624
E	0.7063	0.7126	0.7287	0.7633	0.7929	0.8361	0.8687	0.9136	0.9438	0.9841

REF. ***D.J. Santeler and M.D. Boeckmann, J. Vac. Sci. Technol. A9 (4) p.2378, 1991***

Transmission of Rectangular Ducts – Cont.





Comparing numerically computed transmission with α_s, α_l and corrected α.

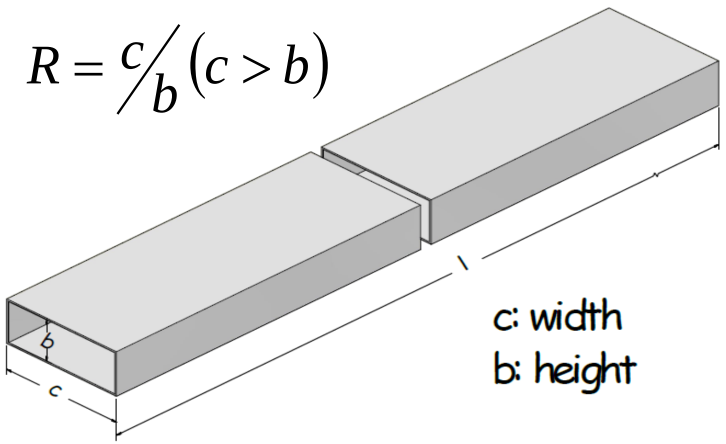
Transmission of Rectangular Ducts – Parameters

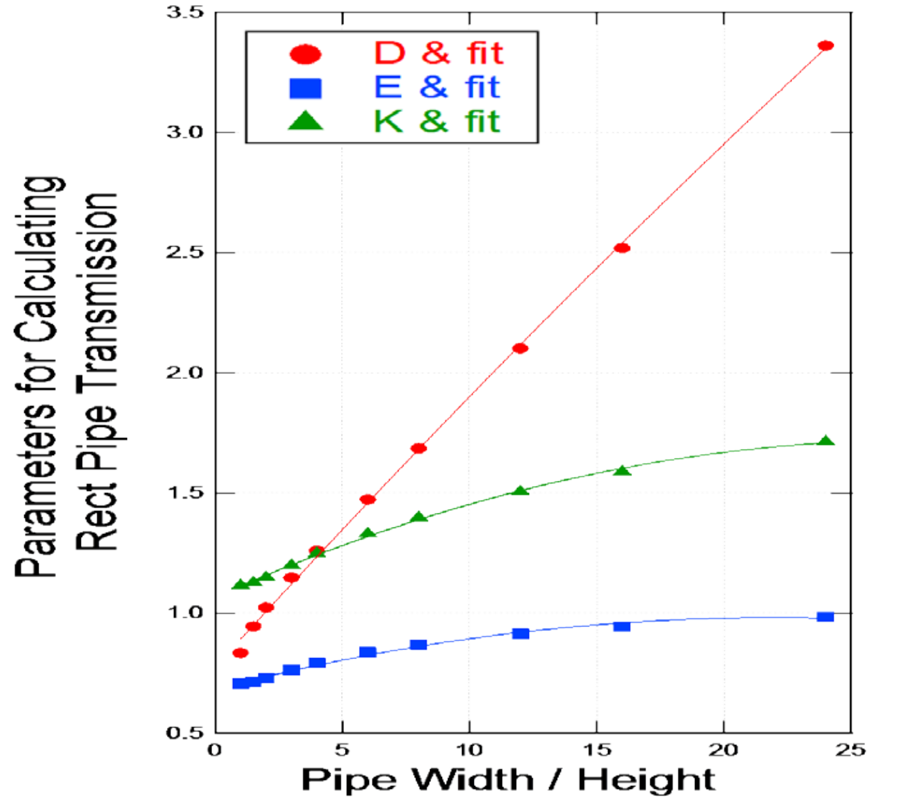
3rd-order polynomial fits are used for any rectangular cross sections (R=c/b as variable):

$$\left(K,D,E\right)=A_0^{(K,D,E)}+A_1^{(K,D,E)}\cdot R+A_2^{(K,D,E)}\cdot R^2$$

Fit	K	D	E
A_0	1.0663	0.7749	0.6837
A_1	0.0471	0.1163	0.02705
A_2	-8.48×10^{-4}	-3.72×10^{-4}	-6.16×10^{-4}

$$R={c}/{b}\ (c>b)$$



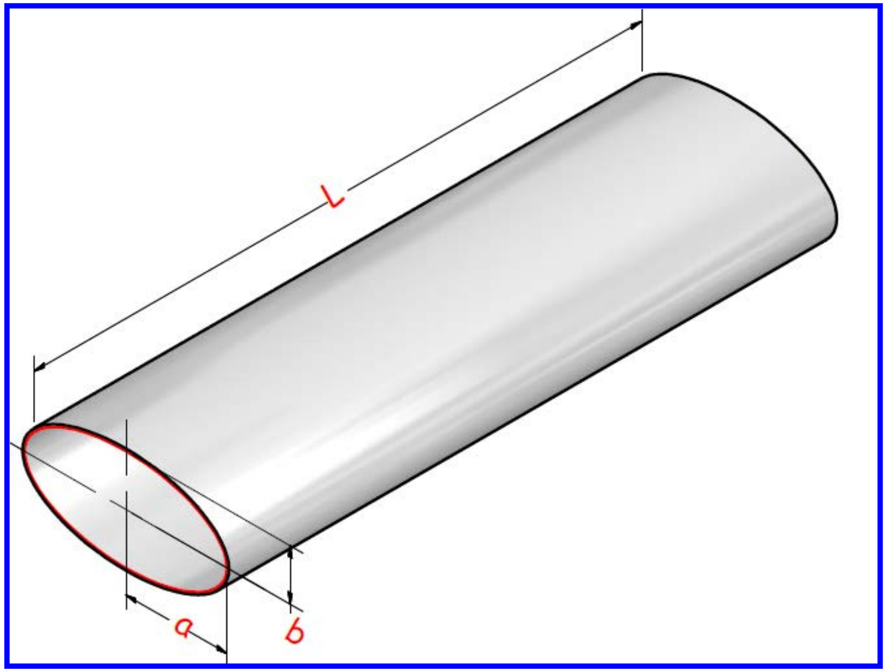


Transmission of Long Elliptical Tubes

*According to Steckelmacher **

$$\alpha_E = \frac{16b}{3\pi L} K(k')$$

where K(k') is complete elliptic integral of 1st kind



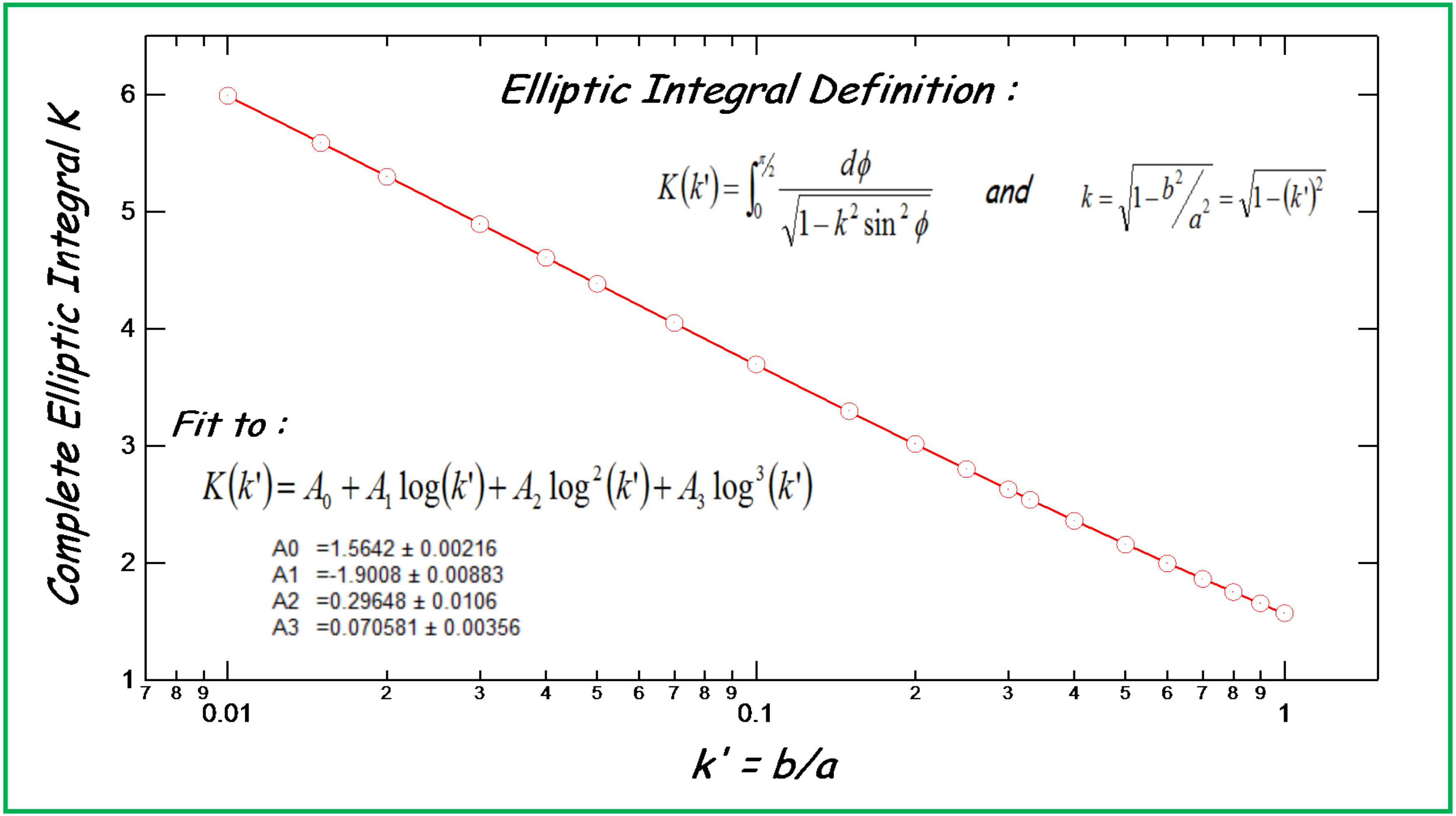
$$K(k') = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2 \sin^2 \phi}}$$

and

$$k = \sqrt{1 - b^2/a^2} = \sqrt{1 - (k')^2}$$

** W Steckelmacher, "Molecular flow conductance of long tubes with uniform elliptical cross-section and the effect of different cross sectional Shapes", J. Phys. D: Appl. Phys., Vol. 11, p.39 (1978).*

Calculating Elliptic Integral



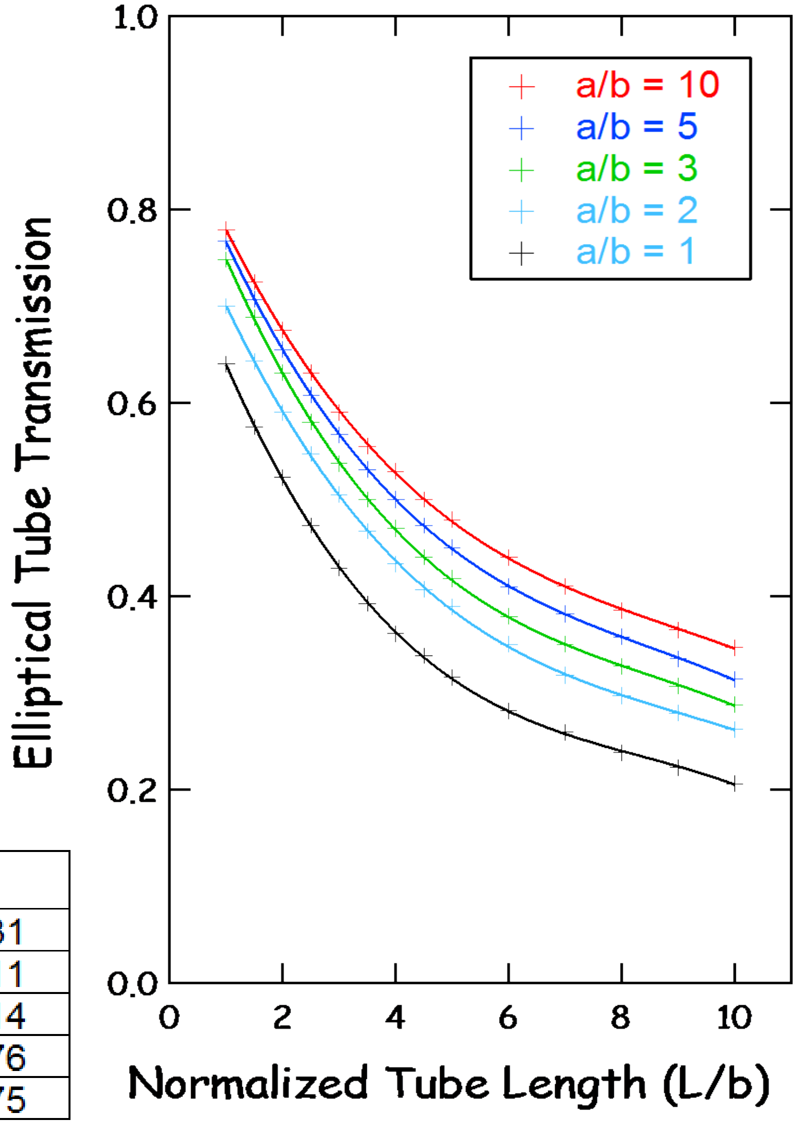
The elliptic integral has to be calculated numerically, but here is a web calculator for the complete elliptic integral of 1st kind.

Transmission of Short Elliptical Tubes

- *T. Xu and J-M. Laurent (CERN-LEP-VA #83-28, 1983) calculated transmission probability of short elliptical tubes, using Monte-Carlo method.*
- *The calculated results are reproduced to the right, with fittings (solid lines) to polynomial function, with fitted coefficients listed in the table below.*

$$\alpha_E = A_0 + A_1(L/b) + A_2(L/b)^2 + A_3(L/b)^3$$

a_vs_b	C_A0	C_A1	C_A2	C_A3
10	0.90689	-0.14103	0.013534	-0.00050431
5	0.90466	-0.15323	0.015408	-0.00060011
3	0.89614	-0.16387	0.016872	-0.00065814
2	0.83579	-0.14943	0.014547	-0.00053476
1	0.79008	-0.16823	0.018372	-0.00072975



Comparing Round, Rectangular and Elliptical Tubes

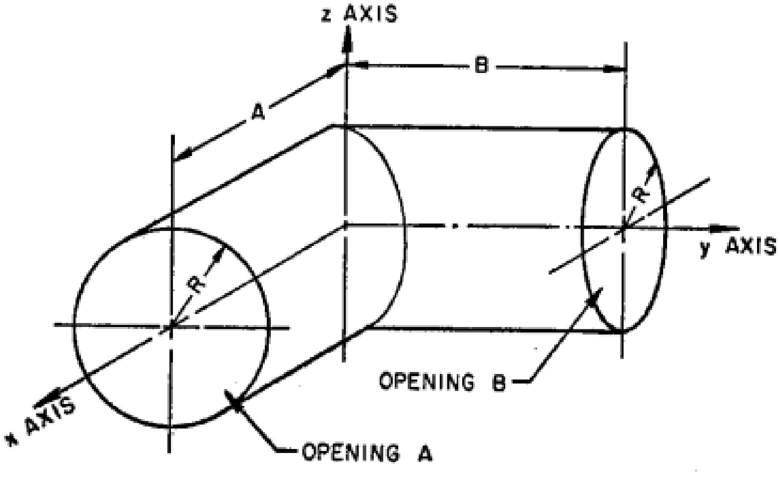
It is interesting to compare molecular transmission probability for tubes with round, rectangular and elliptical cross sections, but with same aperture area (thus the same entrance conductance).

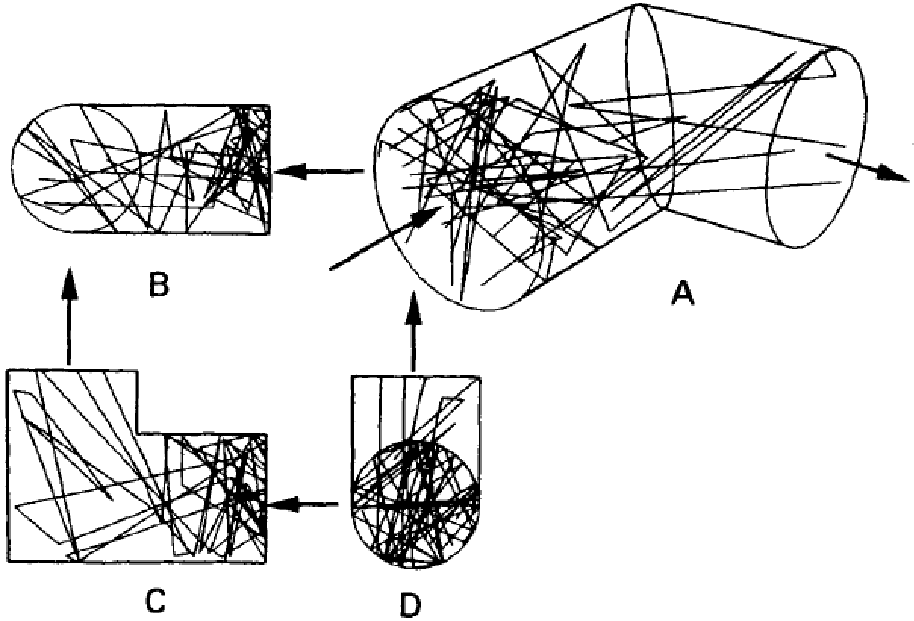
a/b	T_{Ellip}/T_{Round}	T_{Rect}/T_{Round}
1.0	1.0	0.988
2.0	0.971	0.960
3.0	0.931	0.920
4.0	0.892	0.884
5.0	0.859	0.852
10.0	0.744	0.742
20.0	0.624	0.622
100.0	0.381	0.385

For a>>b, both elliptical and rectangular tubes degenerate to slit, so not surprise to see similar transmission probability.

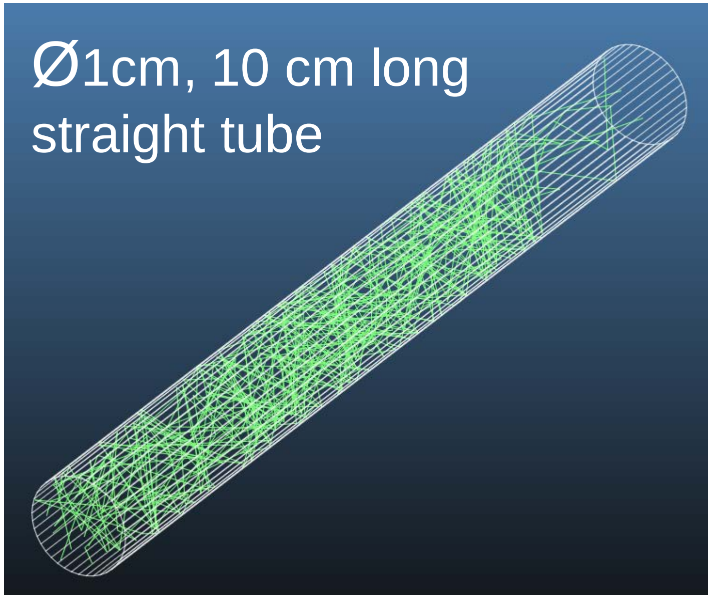
Monte Carlo Test Particle Calculations

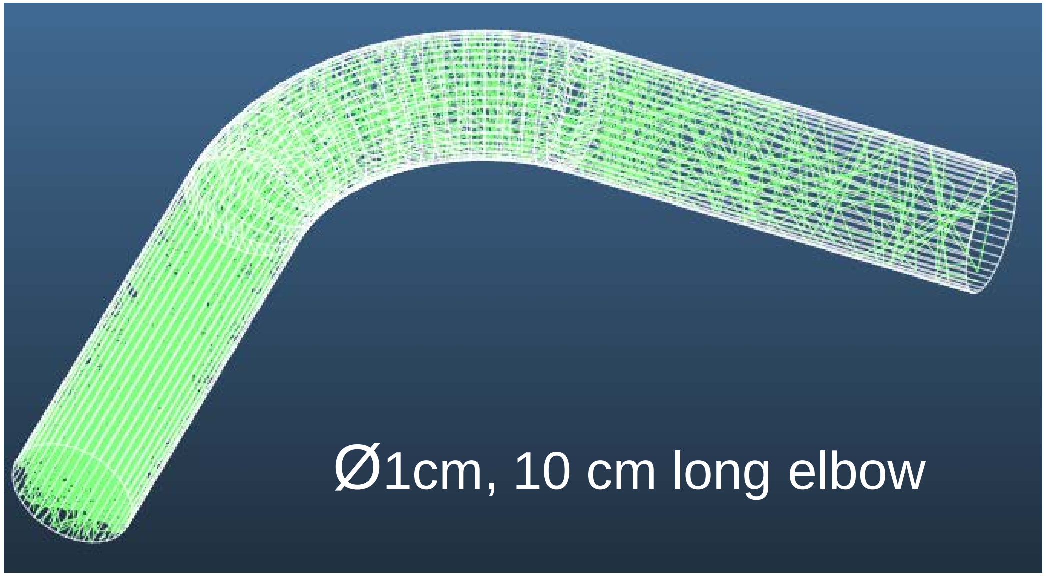
- Monte Carlo (MC) statistic method is often used to calculate complex, but practical vacuum components, such as elbows, baffles, traps, etc.
- In MC calculations, large number of test particles 'injected' at entrance, and tracked through the geometry.
- One of such MC packages, MolFlow+ is available through the authors.



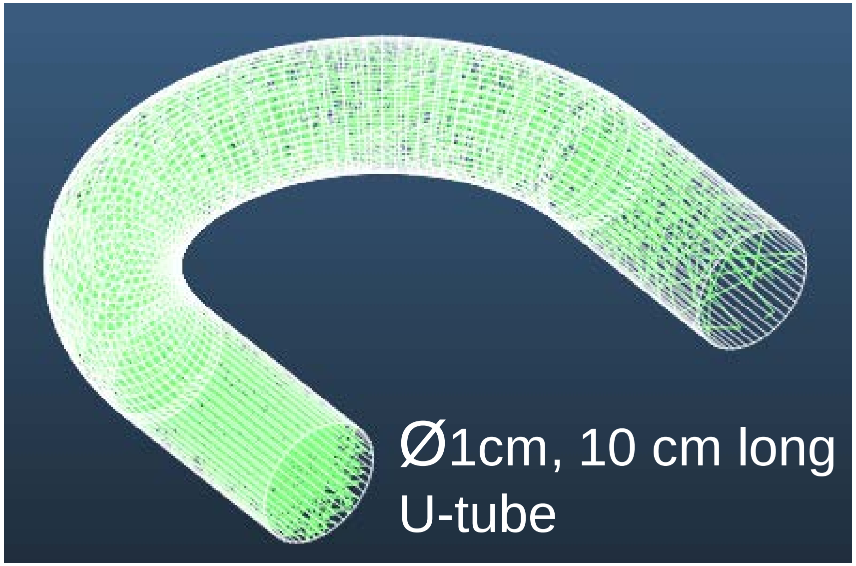


Transmission Calculations Using MolFlow+





Straight	$\alpha = 0.109$
90° Elbow	$\alpha = 0.105$
180° U	$\alpha = 0.093$



These simple calculations took <1-minute

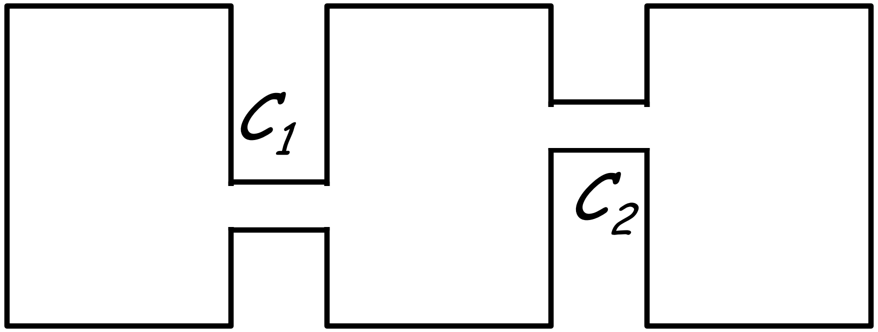
Combining Molecular Conductances

- The conductance of tubes connected in parallel can be obtained from simple sum, and is independent of any end effects.

$$C_T = C_1 + C_2 + C_3 +, \ldots, + C_n$$

- Series conductances of truly independent elements can be calculated:

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_N}$$



The series elements must be separated by large volumes, so that the molecular flow is re-randomized before entering next element